MAINTAINING A CONSISTENT CONTROLLER PERFORMANCE

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ABSTRACT

Classical controller design sought ways to synthesize controllers that satisfied requirements specified in the time domain (such as overshoot or decay ratio) or in the frequency domain (such as gain and phase margins). Quite independently, control auditors used controller performance assessment (CPA) techniques to quantify how close to minimum variance a given loop is actually operating. This paper joins these two tasks. We propose a novel method to design controllers to achieve a specified performance, say 80% of the theoretical optimal control i.e. minimum variance controller. Furthermore the method is adaptive using a recursive least-squares approach to identify the relevant process and noise models, and then solves the Diophantine equation to synthesise the controller with the required properties. This ensures (within reason) that the controller's performance is constant as the plant dynamics slowly change.

Key Words: Controller performance assessment, recursive least squares, time varying system.

INTRODUCTION

In recent years, it has been interesting to note that industrial control practitioners have shifted their focus from designing ever-increasing multi-variable control algorithms to looking more closely at the multitude of existing basic control loops on their plants (Jelali, 2006). This shift is of course motivated by economics since a single poorly performing proportional integral derivative (PID) loop can reverse the gains made by a sophisticated, possibly delicate advanced control strategy. This paper is not suggesting that advanced process control does not have its place, far from it, it is simply suggesting that doing the basics well is an equally profitable exercise.

Undertaking a control audit of the basic control loop is known as control performance assessment, or CPA, and most industrial control system vendors supply software tools to aid this. The key metric that one needs to establish is how close the loops is operating to one that delivers the minimum variance in the output error given a stochastic disturbance. The ratio of the best achievable variance, σ_{MV}^2, to the actual measured variance, σ_y^2, of the controlled variable under assessment,

$$\eta = \frac{\sigma_{MV}^2}{\sigma_y^2} \tag{1}$$

is known as the Harris index, first proposed for linear systems in (Harris, 1989) based on the concept of minimum variance control developed in (Astrom, 1970). This metric can be estimated directly from the measured data.

Linear CPA is routinely applied in the refining, petrochemical, pulp and paper and the mineral processing industries as noted by (Huang and Shah, 1999, Harris, 1999, Jelali, 2006). For loops with significant nonlinear elements, either in the actuator (control valve stiction for example), or the plant dynamics, the strategy to estimate the minimum achievable variance is more complicated as noted by (Yu et al., 2008, Yu et al., 2010) and there is a danger that the performance index is over estimated leaving one in a false sense of satisfaction.

The layout of this paper is as follows. The concept and design of minimum variance controllers are introduced. Next a discussion on how to design a performance based controller from a normal pole placement controller is presented. The next section illustrates the detuning idea which is followed by a discussion and conclusions highlighting both the limitations and potential of the proposed method.

MINIMUM AND NEAR-MINIMUM VARIANCE CONTROL

'Perfect control' is the holy grail of any control engineer, but given limits in actuators and sensors, and characteristics of the plant such as deadtime, it is something one is unlikely to achieve. However we can in principle achieve minimum variance control (Astrom and Wittenmark, 1994), where the variance of the output error of the control loop is at a minimum. For disturbance rejection cases, which form the majority of control loops in the processing industries, this provides a useful benchmark to establish the efficacy of any given control loop. We are not suggesting that minimum variance control is the optimal controller, indeed, it is not in most practical cases, and that precisely is the point of this paper.

Many industrial control loops can be modelled as,

$$\begin{aligned}
A(q)y(t) &= B(q)u(t) + C(q)e(t) \\
A(q) &= q^n + a_1 q^{n-1} + \ldots + a_n \\
B(q) &= b_0 q^{n-d} + b_1 q^{n-d-1} + \ldots + b_m \\
C(q) &= q^n + c_1 q^{n-1} + \ldots + c_n
\end{aligned} \tag{2}$$

where y, u and e are the process output, manipulated input, and noise terms respectively, and in a slight departure from some texts, we are using the *forward* shift operator, q, (for example $q^n y(t)=y(t+n)$) as shown in Figure 1. The variables A, B, C are polynomials in the operator q. We assume polynomials A and C are monic, and that C

has no zeros outside the unit circle. Note that unstable plants (zeros of A outside the unit circle), non-minimum phase plants (zeros of B outside), and plants with deadtime are covered by this framework. The process deadtime, d, is the delay from when a controller output signal is issued until when the controlled variable (CV) first begins to respond.

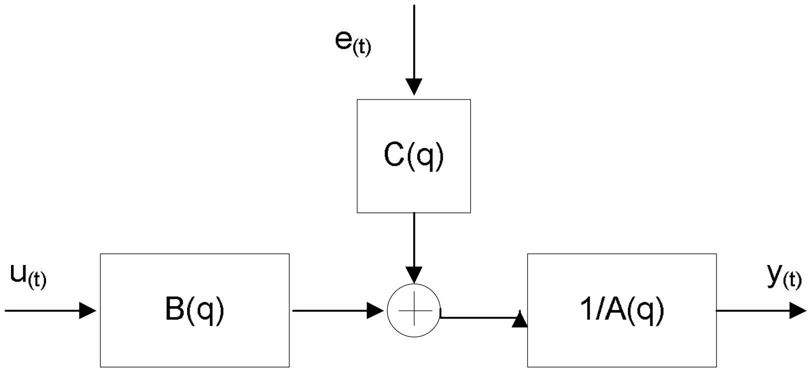

Figure 1: Plant dynamics

We will assume a two degree of freedom controller,

$$F(q)u(t) = -G(q)y(t) + H(q)y^{*}(t) \tag{3}$$

where y^* is the desired setpoint. For disturbance rejection, the main focus of industrial process control, we can, without loss of generality, drop the reference signal y^*, giving the closed loop from noise, $e(t)$, to output $y(t)$, as shown in Figure 2.

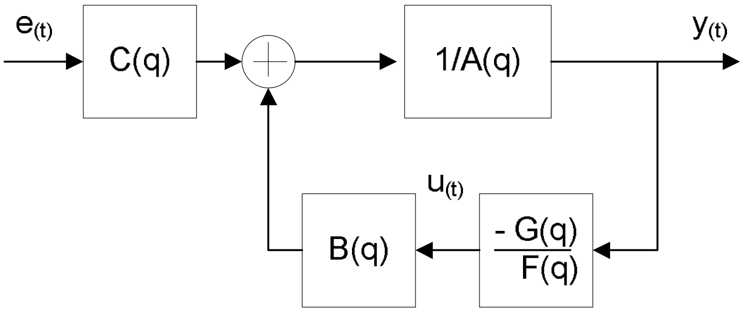

Figure 2: The closed loop from noise, e, to output, y

Following the procedure in (Astrom and Wittenmark, 1994) [pp 143-144], to design the controller polynomials F and G, we desire that the process recovers from a disturbance at most d samples in the future. This being the best we can expect will deliver the minimum output variance and is achieved by solving the Diophantine equation,

$$C = AF + BG \tag{4}$$

Solving Eqn. (4) can be done in a number of ways, but is succinctly expressed in MATLAB notation using matrix convolution as shown in Table 1 (Wilson, 2011).

Table 1: Solving the Diophantine equation

```
function [F,G,Tcheck] = dioph_mtx(A,B,T);
                          % Solves the polynomial Diophantine AF + BG = T for F & G.
da = length(A)-1; db=length(B)-1;
T = [T, zeros(1,da+db-length(T)+1)];      % pad with zeros
dt=length(T)-1;% da = deg(A) etc.
dg =da-1; df=dt-da ;
B = [ zeros(1, df-db+1), B];               % pad with leading zeros
Rac = [convmtx(A,df+1);convmtx(B, dg+1)];
FG = T/Rac;                                % equate coefficients of F & G
F = FG(1:df+1); G = FG(df+2:df+dg+2);      % Note F & G are row vectors.
return
```

Combining the control law, Eqn. (3) with the process, Eqn. (2), gives the closed loop,

$$y(t) = \frac{CF}{AF + GB} e(t) \tag{5}$$

If the controller polynomials are chosen such that Eqn. (3) is a minimum variance controller, then the closed loop is equivalent to the moving average controller,

$$y(t) = F(q)e(t) \tag{6}$$

This makes a convenient check since the cross correlation of the output $y(t)$ will vanish after d lags for a minimum variance controller. For any other controller, the cross-correlation will persist after d lags.

PERFORMANCE BASED CONTROLLER

The idea of performance based control is to identify the poor performance loop first and then to readjust the controller parameters. Mclntosh et al. (2004) implemented this idea using pole placement control and generalized predictive control and Yamanto et al. (2011) developed a performance-based PID controller. In this paper, we will develop a new performance based controller based on pole placement control.

While the minimum variance controller has the attractive property that is an optimal controller, it is also a very aggressive and fragile controller and therefore rarely used as noted in (Zhou et al., 2011). Instead a controller with $\eta \approx 0.8$ is deemed appropriate as opposed to exact minimum variance where $\eta=1$.

Consequently, and as opposed to much of the recent activity which has focussed on establishing a metric for the control performance for a given operating loop, we ask if it is possible to design a controller with a specific Harris index. One way to approach this is to detune a minimum variance controller in some systematic way to a pre-specified η. Since there is an infinite number of admissible controllers for a given η, we could cast this as an optimisation problem where the η requirement is a constraint. While this paper takes a direct design approach, future work will focus on the optimisation problem of given a performance constraint of η^*, we can optimise some other aspect of the control loop such as say minimising the manipulated variable movement to prolong the life of the control valve.

Recall from Eqn. (5) that if the controller polynomials F and G are obtained by solving the Diophantine equation,

$$C = AF + BG \tag{7}$$

then the controller in Eqn. (8) is a minimum variance controller,

$$u(t) = -\frac{G}{F} y(t) \tag{8}$$

If we can include a tuning parameter, $\beta > 1$, into Eqn. (7),

$$\beta C = AF + BG \tag{9}$$

then by substituting Eqn. (9) into Eqn. (5), we will have,

$$\begin{aligned} y(t) &= \beta F e(t) \\ &= \beta(1 + f_1 q^{-1} + \ldots + f_{d-1} q^{-d+1}) e(t) \end{aligned} \tag{10}$$

where the actual variance of output $y(t)$, σ_y^2, is,

$$\sigma_y^2 = \beta^2 (1 + f_1^2 + \ldots + f_{d-1}^2) \sigma_e^2 = \beta^2 \sigma_{MV}^2 \tag{11}$$

where σ_e^2 is the variance of disturbance $e(t)$. The performance index or Harris index is,

$$\eta = \frac{\sigma_{MV}^2}{\sigma_y^2} = \frac{\sigma_{MV}^2}{\beta^2 (1 + f_1^2 + \ldots + f_{d-1}^2) \sigma_e^2} = \frac{1}{\beta^2} \tag{12}$$

The tuning parameter β is equal to,

$$\beta = \sqrt{1/\eta} \tag{13}$$

To illustrate the design procedure, we will assume plant dynamics as,

$$\begin{aligned} B(q) &= -2(q - 0.3)(q - 0.4) \\ A(q) &= (q - 0.9)(q - 0.8)(q - 0.7)(q - 0.2)^2 \end{aligned} \tag{14}$$

with noise polynomial,

$$C(q) = (q - 0.25)^2 \tag{15}$$

and we assume the stochastic variable $e(t)$ is drawn from a normal distribution with zero mean and variance, $\sigma_e^2 = 1$. If we want to find a pole-placement controller providing a

Harris index 0.8, then the tuning parameter β=1.118 can be determined by Eqn. (13). The pole-placement controller is,

$$u(t) = -\frac{G}{F}y(t)$$
$$G = [\text{-}1.6836 \quad 1.9501 \quad \text{-}1.1227 \quad 0.2708 \quad \text{-}0.0222] \tag{16}$$
$$F = [1 \quad \text{-}1.1262 \quad 0.2637]$$

When the process dynamic changes, we can use a recursive least-squares (RLS) approach to identify the new process model, then use proposed method to obtain the performance based controller. This procedure can maintain the process at the user defined performance level i.e. a consistent performance index.

SIMULATION

A Single-Input-Single-Output (SISO) process with time varying disturbance models studied by Olaleye et al. (2004) is used to illustrate the proposed method. The process transfer function is,

$$y(t) = \frac{B}{A}u(t) + \frac{C}{A}e(t) \tag{17}$$

Assume that the process is affected by three different disturbance dynamics as noted in Table 2.

Table 2: Different disturbance dynamics

Sample Period, t	Polynomials
t<2000	$A = [1.0 \quad \text{-}0.67]$ $B = 0.33[0 \quad 0 \quad 0 \quad 0 \quad 1]$ $C = [1 \quad \text{-}0.4]$
$2000 < t < 3000$	$A = [1.0 \quad \text{-}1.67 \quad 0.67]$ $B = 0.33[0 \quad 0 \quad 0 \quad 0 \quad 1 \quad -1]$ $C = [1.0 \quad \text{-}1.07 \quad 0.268]$
$t > 3000$	$A = [1.0 \quad \text{-}1.67 \quad 0.67]$ $B = 0.33[0 \quad 0 \quad 0 \quad 0 \quad 1 \quad -0.87]$ $C = [1.0 \quad \text{-}1.07 \quad 0.268]$

Figure 3 shows the closed loop behaviour using a Dahlin controller given by,

$$\frac{0.5 - 0.9q^{-1}}{0.33 - 0.1q^{-1} - 0.23q^{-2}} \tag{18}$$

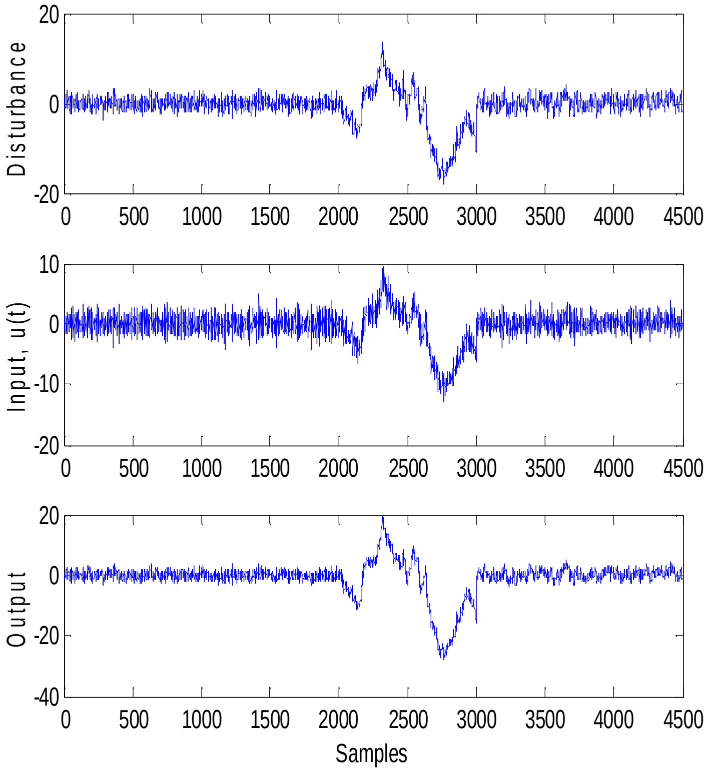

Figure 3: Closed loop behaviour with a Dahlin controller

Figure 4 shows the output under our proposed performance-based adaptive pole - placement controller. The performance indices for both controllers are listed in Table 3.

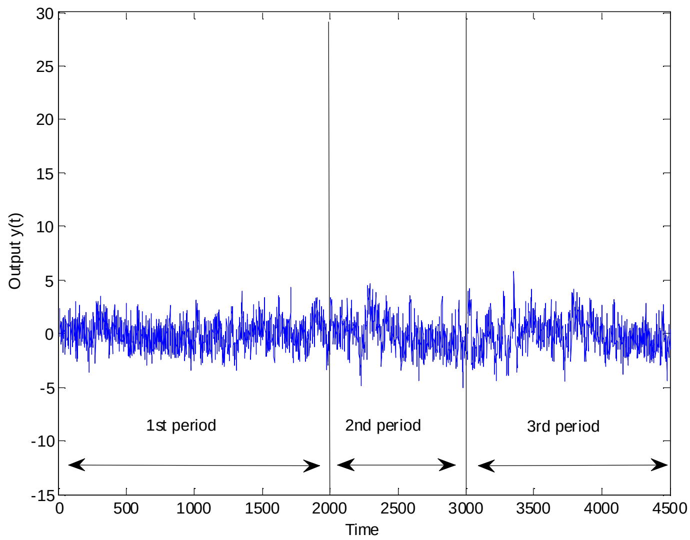

Figure 4: Closed loop output with proposed performance based controller

From Figure 3 and Figure 4, we will observe that the output variance from our proposed performance based controller is less disturbed by different types of disturbances, however, the regular controller i.e. Dahlin controller will provide significant different performances with different disturbances. This conclusion is supported by the Harris indices in Table 3.

Table 3: Harris index for two type controllers

Controller	*Harris Index*		
	Period 1	Period 2	Period 3
Dahlin	0.88	0.02	0.70
Pole-placement	0.8	0.805	0.801

CONCLUSIONS

In this paper, a new performance based controller has been proposed. A connection between the user-specified performance index and the polynomials of pole placement controller is built by adding a tuning parameter β. The simulation results show that the proposed controller can maintain the process at a certain performance level even disturbance dynamic changes. In this paper we only provide one simple way to design the performance based controller, in the future, we will develop more ways to determine the polynomials of pole placement controller.

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BRIEF BIOGRAPHY OF PRESENTER

David Wilson obtained his undergraduate chemical engineering degree from the University of Auckland and Ph.D. from the University of Queensland, Australia in 1990. Since then he has worked at ETH in Zürich, and as a senior lecturer in the Department of Electrical Engineering in Karlstad University, Sweden. Currently he is an Associate Professor in the Department of Electrical and Electronic Engineering at the Auckland University of Technology and co-directs the Industrial Information and Control Centre, (I2C2), a research and consultancy group. He is a corporate member of the Institute of Chemical Engineers (UK), and an Associate editor for the Journal of Process Control. His main research interests are modelling, simulation and advanced control of industrial processes including control performance analysis, high-speed embedded model predictive controllers, and industrial optimisation.