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Control performance assessment in the presence of sampling jitter

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ABSTRACT

Variations in sampling time, or sampling jitter, is a common insidious industrial control problem. This work investigates how the stochastic sampling jitter adversely affects the performance of control loops using control performance assessment (CPA) tools. It is analytically shown that sampling jitter may both improve or deteriorate the control performance, and through simulation, it is demonstrated that the estimates of Harris index from the time invariant ARMA model are reliable, provided the sampling jitter is relatively small. Furthermore we propose a method to analyze the effects of sampling jitter, when the jitter is unknown and not directly measurable. From the simulations of 15 typical processes, it is shown that the proposed method is reliable for most of the investigated cases.

Keywords: Control performance assessment; Sampling jitter; Controller delay; SISO plant models; Time-varying systems

1. Introduction

Control performance assessment, or CPA, is a technology to diagnose and maintain operational efficiency of control systems developed in a direct response to address this increasingly important economic problem. CPA based on linear models is routinely applied in the refining, petrochemicals, pulp and paper and the mineral processing industries as noted by Harris (1999) and Huang and Shah (1999), while a recent practical overview is given in Jelali (2006) and an automated system intended for plant wide use is described in Scali et al. (2009).

While most of the research and commercial activity in CPA has been based on the assumption of a linear plant model to date, those researchers investigating nonlinear systems fall into one of two groups. The first group focused on the diagnosis of a common specific nonlinearity, namely valve stiction (Horch, 1999; Choudhury et al., 2006; Thornhill and Horch, 2007; Choudhury et al., 2007; Yu et al., 2009), culminating in the collection given in Jelali and Huang (2010), while the second group tried to establish the minimum variance performance lower bound (MVPLB) (Harris and Yu, 2007; Zhou and Wan, 2008; Yu et al., 2008, 2009, 2010a,b).

However nonlinearities are not just restricted to the plant or manipulator dynamics, they can stem from the actual system architecture. For example, many real-time systems are implemented as distributed control systems where the digital feedback control of continuous plants is implemented over a communication network or field bus. While the controller supposedly works at a fixed nominal sampling period, T, in many practical cases due to a varying computer load, unmodelled communication delays, this sampling rate can vary, at times significantly. This stochastic variation in the actual T is known as sampling jitter.

Historically the low sampling rates typical in many chemical process control applications have meant that the jitter problem was arguably insignificant and therefore overlooked. However with the increased use of wireless communications between sensors, controllers and actuators (exacerbating the potential jitter problem), and the increased desire to control fast-acting processes such as steam turbines, micro-reactors, and high-speed production such as paper making we believe

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that this will become an important issue in chemical process control. We also note that some techniques popular in chemical process control such as MPC are now finding applications in embedded systems such as position control and robotics, and consequently the division between chemical process control and mechatronics for example will become increasingly blurred in the near future. While this paper is restricted to SISO systems, the jitter problem is more likely in computationally heavy multivariable applications.

Systems exhibiting irregular sampling rates due to jitter have been addressed in many research areas such as signal process and control. Jitter detection and measurement were addressed in Liu (1992) and Sharfer and Messer (1994) and jitter modeling and identification have been studied in Ou et al. (2004), Boje (2005), Eng and Gustafsson (2008), and Burnham et al. (2009). The topics of controlling systems with jitter problems can be found in Sala (2005) and Nilsson et al. (1997), and a dedicated jitter compensation controller was discussed in Lincoln (2002) and Niculae et al. (2008). In this paper, we will study the control performance assessment problem for linear systems experiencing sampling jitter. While we are looking at the effect of jitter on the control performance, Thornhill et al. (2004) studied the related problem of excessive data compression on computing the CPA.

We should point out that the focus of this research is significantly different to that reported in Huang (2002) and Olaleye et al. (2004) even though context is superficially similar. In those reports, the CPA was addressed to the time-varying systems where the time-varying coefficients can be expressed as known functions of time. In this paper the coefficients of the discretised plant model are stochastic variables caused by the random sampling period.

Formally to estimate the performance index, the time varying ARMA model must be estimated which requires the knowledge of the unobservable jitter magnitude. In this paper, we proposed a method to evaluate the jitter effect on the control performance from the observable operating data.

The layout of the paper is as follows. Section 2 introduces the linear time varying system (LTV) structure studied in this paper. In Section 3, control performance assessment is briefly introduced and the minimum variance performance lower bounds and performance index are reviewed. In Section 4, minimum variance control and the performance index are addressed for LTV systems, and the jitter sampling effect on the control performance is discussed. The estimation of performance index and the sampling jitter effect are also discussed. In Section 5, simulation examples are used to illustrate the proposed methods. It is followed by a discussion and conclusions highlighting both the limitations and potential of the proposed methods.

2. Process formulation

In the following development we must make some assumptions regarding the control system of interest, specifically the shape of the probability distribution (PDF) of the sampling jitter. The obvious PDF forms are Gaussian about some nominal sample rate or uniform, but in many cases it is may be more realistic to have a bimodal distribution, or at least a distribution with a significant right-hand tail. The motivation for these types of distributions is due to the fact that at the times when significant control calculations are needed is when good control is also needed. Some controllers with a constant structure and operation count (such as PID) are independent of state (except when transferring from auto to manual), while others such as MPC take longer to solve the embedded optimisation problem when recovering from upsets, or moving to a new setpoint. The embedded MPC controller in Currie and Wilson (2010) illustrates this by taking 5 times as long when solving the constrained optimisation problem compared to when the unconstrained controller is active. Finally in many instances a single chip will be controlling a number of loops, so a time out in any one loop will cause interrupts to be missed in the others.

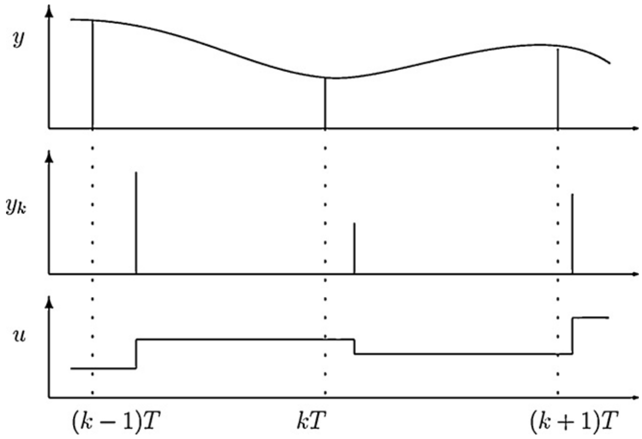

Fig. 1 – Signals of a control system with the sampling jitter problem.

For the remainder of this paper, we will assume uniform sampling with jitter, where the sampled signal, t_k, are taken at time moments $t_k = t_{k-1} + (T + \tau_k)$, where T is the nominal sampling interval, and the jitter component, τ_k, is a family of identically distributed independent random variables, $\tau \in \mathcal{U}(0, \tau_{max})$.

We will make the following assumptions about the control system: The timing of the process output measurements are disturbed by sampling jitter. The control signal is applied to the process as soon as the data arrives. The jitter sampling period $T + \tau_k$ is always great than, or equal to, the constant sampling period T when there is no jitter in the control system. Consequently we assume that the random sampling period $T + \tau_k$ is uniformly distributed in interval $[T, T_{max}]$. The third assumption that sampling with jitter always causes a *late* control signal is based on how real-time controllers are implemented. Since the microcontroller is interrupt driven, when the clock sends the interrupt signal, the micro responds with the most recently calculated manipulated variable signal. The micro will never respond faster than the accurate clock signal, but could well fail to service the interrupt in time, hence be late.

The timing of signals in the control system with the sampling jitter problem is illustrated in Fig. 1. In Fig. 1, the first diagram shows the process output and the samples with a constant sampling period T, the second diagram illustrates the exact signal recorded with the jitter sampling situation, the third diagram shows the process inputs.

Given a single-input single-output (SISO) continuous time (CT) system

$$y_{(t)} = G(s)u_{(t)} \tag{1}$$

where for simplicity $G(s)$ has n distinct poles,

$$G(s) = \sum_{i=1}^{n} \frac{c_i}{s-\lambda_i} e^{-\theta s} \tag{2}$$

where we sample at instants $t_k \in \mathbb{R}$, $k = 0, 1, \ldots$, with $t_{k+1} > t_k$, $t_0 = 0$. The time-varying sampling period will be denoted by $T_k = T + \tau_k$, and it will be assumed to lie in a known compact set $\mathfrak{J}$. Given the output at time t_k, $y(t_k)$, if the input $u(t_k)$ is kept constant across the inter-sampling interval, the output at time t_{k+1} is,

$$y(t_{k+1}) = G(q^{-1})q^{-f+1}u(t_k) \tag{3}$$

where q^{-1} is the backshift operator, $q^{-1}y_{k+1} = y_k$.

The discrete transfer function is

$$G_{T_k}(q^{-1}) = \sum_{i=1}^{n} c_i \frac{e^{\lambda_i T_k}}{\lambda_i} \cdot \frac{1}{1-e^{\lambda_i T_k} q^{-1}} \tag{4}$$

Introducing the notation $G_k(q^{-1}) = G_{T_k}(q^{-1})$, $y(t_{k+1}) = y_{k+1}$ and $u(t_k) = u_k$, the system in Eq. (3) can be succinctly expressed as,

$$\begin{aligned} y_{k+1} &= G_k(q^{-1})q^{-f+1}u_k \\ &= \frac{B_k(q^{-1})}{A_k(q^{-1})} q^{-f+1} u_k \end{aligned} \tag{5}$$

where $A_k(q^{-1})$ and $B_k(q^{-1})$ are time-varying polynomials (due to the time-varying sampling rate), and f is the time delay of the system which is assumed known. If there is an additive disturbance d_k, the total process can be written as,

$$y_k = \frac{B_k(q^{-1})}{A_k(q^{-1})} q^{-f} u_k + d_k \tag{6}$$

where d_k can be modelled as the output of a linear Autoregressive-Integrated-Moving-Average (ARIMA) filter driven by white noise a_k of zero mean and variance σ_a^2 of the form

$$d_k = \frac{\theta_k(q^{-1})}{\phi_k(q^{-1})\nabla^h} a_k \tag{7}$$

where $\nabla = (1 - q^{-1})$ is the difference operator and h is a non-negative integer, typically less than 2. The polynomials $\theta_k(q^{-1})$ and $\phi_k(q^{-1})$ are monic and stable. The control performance assessment for the more general LTV SISO in which the disturbance d_t is also time varying can be found in Huang (2002). Finally in this paper we will assume that the process time delay, being a function of the plant only (as opposed to a combination of plant and measurement dynamics) is constant and independent of sampling jitter. The significance of this assumption will be explored in subsequent research.

3. Control performance assessment for linear time-invariant systems

Control performance assessment (CPA) is widely applied in the refining, petrochemicals, pulp and paper and the mineral processing industry to diagnose and maintain operational efficiency as reviewed in Jelali (2006), Qin (1998), Harris et al. (1999), and Huang and Shah (1999). CPA methods have formed a basis for many industrial control performance assessment applications (Huang and Shah, 1999; Harris et al., 1996; Kozub, 1996; Thornhill et al., 1999; Hoo et al., 2003).

Although descriptive statistics such as the mean and variance of manipulated and controlled variables and the percentage of constraints occurrence time can be used as a simple tool for CPA, a comprehensive approach for control performance assessment usually includes the following steps (Jelali, 2006; Harris et al., 1999): (i) determination of a benchmark for control performance assessment, (ii) detection of the poor performing loop, (iii) diagnosis of the underlying causes for poor performance, and (iv) suggestion of improvement.

The first control performance benchmark was proposed by Harris (1989) and is the so called minimum variance performance lower bound (MVPLB). It is based on minimum variance control (Åström, 1970; Box and Jenkins, 1970) where the controller performance index is the ratio of the best achievable variance to the actual measured variance of the controlled variable under assessment. Furthermore, Harris (1989) was first to propose that these performance indices can be estimated directly from the routing operating data by fitting the controlled variable into a ARIMA time series model.

3.1. *Minimum variance performance lower bounds*

If the sampling period is constant, the linear time-varying (LTV) system in Eq. (6) becomes a linear time-invariant (LTI) system as,

$$y_k = \frac{B(q^{-1})}{A(q^{-1})} q^{-f} u_k + d_k \tag{8}$$

$$d_k = \frac{\theta(q^{-1})}{\phi(q^{-1})\nabla^h} a_k \tag{9}$$

The minimum variance control of the above LTI system first derived by Åström (1970) and Box and Jenkins (1970) is feedback control which achieves minimum output variance. When the model in Eqs. (8) and (9) is known, a controller can be designed to minimize the variance of output y_k. To derive the minimum variance controller, we need to know the f-step ahead minimum-mean-square-error forecast for y_{k+f}:

$$\begin{aligned} y_{k+f} &= \frac{B(q^{-1})}{A(q^{-1})} u_k + d_{k+f} \\ &= \frac{B(q^{-1})}{A(q^{-1})} u_k + d_{k+f|k} + e_{k+f|k} \\ &= y_{k+f|k} + e_{k+f|k} \end{aligned} \tag{10}$$

The variables $d_{k+f|k}$ and $y_{k+f|k}$ are the f-step ahead minimum-mean-square-error forecast for the disturbance and y_{k+f}, respectively; and $e_{k+f|k}$, the prediction error, is a moving average process of order $f-1$ as:

$$e_{k+f|k} = \left(1 + \psi_1 q^{-1} + \ldots + \psi_{f-1} q^{-(f-1)}\right) a_{k+f} \tag{11}$$

The φ weights are identical to the first $f-1$ impulse coefficients of the disturbance transfer function, Eq. (9).

The first term on the right-hand side in Eq. (10), $y_{k+f|k}$, depends on data up to sample k, while the second term depends only on data after time k. Therefore, no matter what controller is used, the two terms are independent.

The control signal which results in the minimum achievable variance in the output can be obtained by solving the following relation:

$$\frac{B(q^{-1})}{A(q^{-1})}u_t + d_{k+f|k} = 0 \tag{12}$$

Therefore, The process output under minimum variance control, y_{k+f}^{MV}, will depend on only the most recent f past disturbances, i.e.,

$$y_{k+f}^{MV} = e_{k+f|k} \tag{13}$$

The key observation is that no matter which feedback controller is used, as long as closed-loop stability is preserved, the prediction error, $e_{k+f|k}$ are unaffected. Since it does not depend on the manipulated variable over the prediction interval $k = 1, \ldots, f$, it is called feedback invariant. This point is very important in the study of the minimum variance performance bound. The minimum variance performance lower bound (MVPLB), as measured in the mean square sense, can be written as:

$$\sigma_{MV}^2 = \text{var}\left\{y_{k+f}^{MV}\right\} = \left(1 + \psi_1^2 + \ldots + \psi_{f-1}^2\right)\sigma_a^2 \tag{14}$$

The popular performance index, the Harris index, can be calculated as

$$\eta = \frac{\sigma_{MV}^2}{\sigma_y^2} \tag{15}$$

3.2. *Estimation of the minimum variance performance bound*

If the process is controlled by a linear feedback controller, in the case of linear process with additive disturbances, Harris (1989) has shown that the lower bound on performance can be estimated from routine operating data. The theoretical minimum variance lower bound on performance can be estimated by the following methods:

1. Indirect estimation
   (a) Using an ARIMA model (Harris, 1989): A time series model of the form

   $$\begin{aligned} y_k &= \frac{\alpha(q^{-1})}{\beta(q^{-1})}a_k \\ &= \left(1 + \hat{\psi}_1 q^{-1} + \ldots + \hat{\psi}_{f-1}q^{-(f-1)} + \ldots\right)a_k \end{aligned} \tag{16}$$

   is identified from a set of observations of y. The data can be collected with or without feedback control. The only requirement for the data is that it contains a representative sample of process disturbances. The first $b-1$ impulse coefficients of $(\alpha(q^{-1}))/(\beta(q^{-1}))$ are estimates of the first $b-1$ coefficients of the open-loop disturbance transfer function. The estimate of the achievable minimum variance can be calculated from the estimates of the first $f-1$ coefficients and the residual variance, $\hat{\sigma}_a^2$. That is:

   $$\hat{\sigma}_{MV}^2 = \left(1 + \hat{\psi}_1^2 + \ldots + \hat{\psi}_{f-1}^2\right)\hat{\sigma}_a^2 \tag{17}$$

   (b) Using a Laguerre network (Lynch and Dumont, 1996): in this method, Laguerre networks are used to model the output instead of the ARIMA models. The detailed algorithm can be found in Lynch and Dumont (1996).
2. Direct estimation (Desborough and Harris, 1992): fitting a time series model of the form:

   $$y_{k+f} = \xi(q^{-1})y_k + \varepsilon_{k+f} \tag{18}$$

   from routine open-loop/closed-loop data. With this approach the variance of the residual $\varepsilon(k+f)$ in Eq. (18) is an estimate of σ_{MV}^2.

With above methods, a number of performance indices can be estimated. They can used at the design stage to compare the performance of different controllers. However, they are often used as part of a monitoring and diagnosis scheme for control systems. The methodology has been extended to SISO non-minimum-phase systems (Tyler and Morari, 1995), time variant systems (Huang, 2002) and multivariate systems (Harris et al., 1996; Huang et al., 1997). Readers are referred to Jelali (2006) for recent reviews and a number of extensions, modifications and applications.

4. Control performance assessment for the jitter cases

4.1. *Minimum variance lower bound with jitter cases*

Firstly, let us obtain the minimum variance performance lower bound (MVPLB) for the time-varying system in Eq. (6). We only need to show that the f-step ahead prediction error, $e_{k+f|f}$, is independent of the manipulated variable action. The feedback invariant is given in

$$y_{k+f} = \frac{B_{k+f}(q^{-1})}{A_{k+f}(q^{-1})}u_k + d_{k+f|k} + e_{k+f|k} \tag{19}$$

$$= y_{k+f|k} + e_{k+f|k} \tag{20}$$

where

$$e_{k+f|k} = \left(1 + \varphi_1^{k+f}q^{-1} + \ldots + \varphi_{f-1}^{k+f}q^{-(f-1)}\right)a_{k+f} \tag{21}$$

and the φ weights are the impulse coefficients of the $(\theta(q^{-1}))/(\phi(q^{-1})\nabla^h)$ transfer function and

$$\begin{aligned} y_{k+f|k} &= \frac{B_{k+f}(q^{-1})}{A_{k+f}(q^{-1})}u_k + \frac{P_f(q^{-1})}{\phi_{k+f}(q^{-1})\nabla^h}a_k \\ &= \frac{B_{k+f}(q^{-1})}{A_{k+f}(q^{-1})}u_k + \frac{P_f(q^{-1})}{\phi_{k+f}(q^{-1})\nabla^h}(y_k - \hat{y}_{k|k-1}) \end{aligned} \tag{22}$$

$P_f(q^{-1})$ is a polynomial obtained by solving the Diophantine equation:

$$\frac{\theta_{k+f}(q^{-1})}{\phi_{k+f}(q^{-1})\nabla^h} = 1 + \varphi_1^{k+f}q^{-1} + \ldots + \varphi_{f-1}^{k+f}q^{-f+1} + q^{-f}\frac{P_f(q^{-1})}{\phi(q^{-1})\nabla^h} \tag{23}$$

For the system under minimum variance control, the terms $((B_{k+f}(q^{-1}))/(A_{k+f}(q^{-1})))u_k + d_{k+f|k}$ in Eq. (19) equal zero, and y_{k+f}^{MV} can be written as,

$$y_{k+f}^{MV} = e_{k+f|k} \tag{24}$$

The MVPLB in the mean square sense can then be expressed as,

$$\sigma_{MV}^2 = \text{var}\left\{y_{k+f}^{MV}\right\} = \left(1 + \left(\psi_1^{k+f}\right)^2 + \ldots + \left(\psi_{f-1}^{k+f}\right)^2\right)\sigma_a^2 \quad (25)$$

4.1.1. Estimating the MVPLB in the case of Jitter

If the process in Eq. (24) is controlled by a linear feedback controller of the form $u_k = \gamma(q^{-1})(y_k - y_{sp})$, then the conditional prediction error $e_{k+f|k}$ is feedback invariant and can theoretically be recovered from routine operating data (Huang, 2002; Harris, 1989). This can be done by estimating the f step ahead prediction of output, $y_{k+f|k}$, which can be expressed in form of a time varying ARMA model,

$$\tilde{A}_{k+f}(q^{-1})y_{k+f|k} = \tilde{B}_{k+f}(q^{-1})\left(\gamma(q^{-1})(y_k - y_{sp})\right) + \tilde{C}_{k+f}(q^{-1})y_k \quad (26)$$

where

$$\tilde{A}_{k+f}(q^{-1}) = \theta_{k+f}(q^{-1}) \quad (27)$$

$$\tilde{B}_{k+f}(q^{-1}) = \phi\nabla^h B_{k+f}(q^{-1})\left(1 + \varphi_1^{k+f}q^{-1} + \ldots + \varphi_{b-1}^{k+f}q^{-(b+1)}\right) \times A_{k+f}(q^{-1}) \quad (28)$$

$$\tilde{C}_{k+f}(q^{-1}) = P_b(q^{-1}) \quad (29)$$

The identification and estimation of this time varying ARMA model is a complex task which usually needs some prior information. Consequently in this paper, we will simply use the time invariant ARMA model to fit the outputs from the time varying system described in Eq. (6). Through simulations of 15 typical plant models spanning a wide range of dynamic characteristics, we will investigate the feasibility of this method. If we ignore the presence of the sampling jitter, and compute the MVPLB in standard manner (incorrectly) assuming a time-invariant linear system, we will incur a bias. To show this, let $\tilde{A}$, $\tilde{B}$ and $\tilde{C}(q^{-1})$ be the estimates of polynomials $\tilde{A}_{k+f}$, $\tilde{B}_{k+f}$ and $\tilde{C}_{k+f}$ of the true time-varying ARMA model in Eq. (26),

$$\tilde{A}(q^{-1})\hat{y}_{k+f|k} = \tilde{B}(q^{-1})\left(\gamma(q^{-1})(y_k - y_{sp})\right) + \tilde{C}(q^{-1})y_k \quad (30)$$

Let ϵ_k denote the bias between $\hat{y}_{k+f|k}$ and $y_{k+f|k}$, and ϵ_k is uncorrelated with the disturbance a_{k+i}, $i = 1, \ldots, f$. The f-step ahead prediction can be written as,

$$\hat{y}_{k+f|k} = y_{k+f|k} + \epsilon_k \quad (31)$$

By subtracting y_{t+b} from both sides of Eq. (31), we will have the estimate of the f-step ahead prediction error, $\hat{e}_{k+f|k}$, as

$$\hat{e}_{k+f|k} = e_{k+f|k} + \epsilon_k \quad (32)$$

and since ϵ_k and $e_{k+f|k}$ are uncorrelated, the estimate of the MVPLB can be calculated as

$$\hat{\sigma}_{MV}^2 = \sigma_{MV}^2 + \text{var}\{\epsilon_k\} \quad (33)$$

thus showing that the estimate of the MVPLB and consequently the performance index using linear CPA techniques will be over-estimated.

4.2. *Sampling jitter effect on control performance*

Since the sampling jitter will affect the coefficients of the polynomials $A_k(q^{-1})$ and $B_k(q^{-1})$ in Eq. (6), the variance of output y_k will also be affected. Let us assume that the setpoint is zero and with a certain linear controller $u_k = -((\alpha(q^{-1}))/(\beta(q^{-1})))y_k$, the closed-loop transfer function of the control system in Eq. (6) can be expressed as,

$$y_k = \frac{A_k(q^{-1})}{B_k(q^{-1})}q^{-f}\left(-\frac{\alpha(q^{-1})}{\beta(q^{-1})}y_k\right) + d_k \quad (34)$$

Rearranging Eq. (34), we will have,

$$y_k = \frac{B_k(q^{-1})\beta(q^{-1})d_k}{A_k(q^{-1})\alpha(q^{-1}) + q^{-f}B_k(q^{-1})\beta(q^{-1})} \quad (35)$$

$$= \left(\psi_0^k + \psi_1^k q^{-1} + \psi_2^k q^{-2} + \ldots\right)a_k \quad (36)$$

where $\psi_0^k = 1$.

The variance of the output y_k, $V(y_k)$, can be obtained from,

$$V(y_k) = \sum_{i=0}^{\infty}\left(\psi_i^k\right)^2\sigma_a^2 \quad (37)$$

In this paper, we call $\sum_{i=0}^{\infty}(\psi_i^k)^2$ the sum of square weights of jitter effect (SSWJE). Let y_k^* denote the output of the control loop without jitter sampling and T denote the constant sampling period, the output y_k, can be expressed as,

$$y_k^* = \frac{B(q^{-1})\beta(q^{-1})d_k}{A(q^{-1})\beta(q^{-1}) + q^{-f}B(q^{-1})\alpha(q^{-1})} = \left(\psi_0 + \psi_1 q^{-1} + \psi_2 q^{-2} + \ldots\right)a_k \quad (38)$$

where $\psi_0 = 1$, and

$$G(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})} = \sum_{i=1}^{n} c_i\frac{e^{\lambda_i T}}{\lambda_i}\cdot\frac{1}{1 - e^{\lambda_i T}z^{-1}} \quad (39)$$

The variance of the output y_k^* without sampling jitter, $V(y_k^*)$, can be obtained as,

$$V\left(y_k^*\right) = \sum_{i=0}^{\infty}\psi_i^2\sigma_a^2 \quad (40)$$

It is obvious that the variance of y_k is affected by the time-varying polynomials $A_k(q^{-1})$ and $B_k(q^{-1})$ which in turn are directly caused by the sampling jitter. In the cases where $V(y_k)$ is less than $V(y_k^*)$, the sampling jitter actually helps the control performance, so reducing or eliminating the sampling jitter will not provide any improvement. In the cases where $V(y_k) > V(y_k^*)$, the sampling jitter adversely effects the control performance, so it behooves one to address this.

To estimate the effects of jitter on the control performance for routine operating data, we need to know the process model, controller and disturbance model in Eq. (35). However given that we only have access to the sampled input/output data, we will identify a discrete Box–Jenkins model and use that for the performance index calculation. The CPA procedure in cases where significant sampling jitter is suspected is:

(i) A Box–Jenkins model is identified from the input/output data assumed at nominal sampling time T;
(ii) A continuous process model is approximated from the discrete Box–Jenkins model obtained in step (i);
(iii) Various discrete process models are generated using different sampling periods;
(iv) The impulse weights in Eq. (36) are calculated for each of the discrete process models generated in step (iii), and subsequently the sum of the square weights of jitter effect (SSWJE).

The procedure is illustrated in Fig. 2.

If SSWJE increases rapidly while the sampling period increases, the jitter will significantly adversely effect the control performance. The jitter problem must then be addressed in order to improve the controlled performance. Conversely, if the SSWJE increases only slowly or decreases while the sampling period increases, control engineers do not need to consider the jitter problem for the control performance improvement.

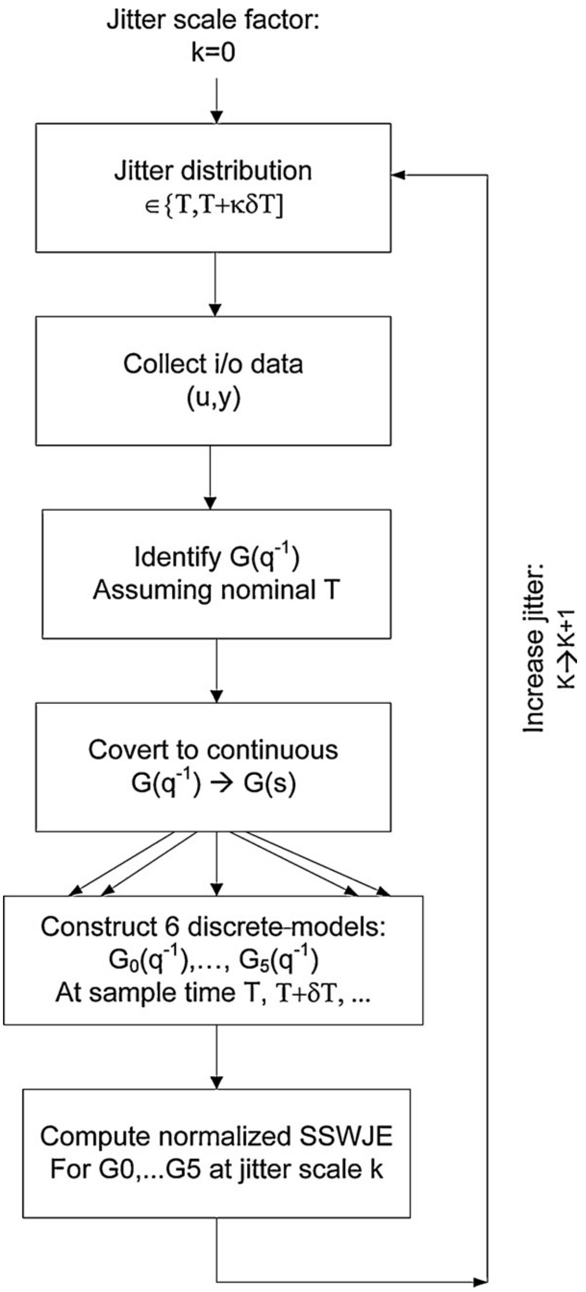

Fig. 2 – Estimating the jitter effect on control performance.

5. Simulation experiments

In this section, the proposed method will be applied to a collection of dynamic systems that together represent the behaviour commonly encountered in the process industries. Fifteen different processes (P1–P15) collected from the test in set provided in Hägglund and Åström (2002) and added to in Skogestad (2003) but with additional delay (specified in units of nominal sample time T) are listed in Table 1. Note that some of the processes are unstable and non-minimum phase but as noted in Tyler and Morari (1995) and Kammert et al. (2003) the MVPLB is the same form as Eq. (14) so our proposed method is still applicable. The tuning of the PI controllers of these modified plants follows the well-regarded algorithm given in (Skogestad, 2003).

Three additive ARMA disturbance models were also considered:

$$\text{Noise Model 1:} \quad d_k = \frac{a_k}{1-0.8q^{-1}} \tag{41}$$

$$\text{Noise Model 2:} \quad d_k = \frac{a_k}{1+0.8q^{-1}} \tag{42}$$

$$\text{Noise Model 3:} \quad d_k = \frac{a_k}{1-1.6q^{-1}+0.8q^{-2}} \tag{43}$$

where a_k is a sequence of i.i.d. Gaussian random noise with zero mean and constant variance $\sigma_a^2 = 0.01$. Note that disturbance model 2 has a pole at $q = -0.8$ contributing to an oscillatory noise sequence. These three disturbance models are randomly assigned to each process model listed in Table 1. In all cases the sampling jitter is assumed uniformly distributed across the interval $[T, T_{max}]$, where T_{max} is around 2–3 times the nominal sampling time, T.

First consider process model 1 with disturbance model 1 with the sampling period ranging between T and $2T$ as shown in Fig. 3. The estimated performance indices for different values of T_{max} are plotted in Fig. 4 where □ are the true performance indices, and the statistical box plots are the estimates using the method outlined in Section 4.1.1.

It is evident in Fig. 4 that the performance index decreases while exhibiting a growing uncertainty and bias as the value of T_{max} increases. Consequently a simple estimation strategy using time-invariant ARMA model provides reliable results only within a relatively small variation of sampling jitter. If one suspects that the magnitude of the jitter is significant, then a time-varying ARMA model must be identified.

The estimates of jitter effects on the control loop on P1 and P3 are plotted in Fig. 5(a) and (b) respectively. The curves in Fig. 5 are obtained in two steps.

For a given amount of jitter, k, spanning from no jitter, ($T_{max} = T$) to considerable jitter, $k = 5$, or $T_{max} = T + k\delta T$, we generate some input/output data. Using this irregularly sampled data, we estimate a discrete Box–Jenkins model, $G(q^{-1})$, with nominal sampling period T. That is, the model is known to be wrong, given that it assumes (incorrectly) a regular sampling time. This model is then converted to continuous time, $G(s)$, then discretised back again at six different sampling times: T, $T + \delta T$, $T + 2\delta T$, ..., $T + 5\delta T$. The normalised SSWJE is calculated for each of these six models, and plotted against the amount of jitter k.

Table 1 – Test plant models used for the simulation experiments showing the PI tuning, the nominal sampling time, the jitter range and the additional number of samples delay.

Case	Process model	Noise model	PI controller		T	T_{max}	Process delay, bT
			K_c	τ_I			
P1	$\frac{1}{(s+1)(0.2s+1)}$	1	2.75	1.1	0.05	0.1	2
P2	$\frac{(-0.3s+1)(0.08s+1)}{(2s+1)(s+1)(0.4s+1)(0.2s+1)(0.05s+1)^3}$	1	0.706	2.5	0.1	0.2	3
P3	$\frac{2(15s+1)}{(20s+1)(s+1)(0.1s+1)^2}$	2	1	1.05	0.1	0.2	2
P4	$\frac{1}{(s+1)^4}$	3	0.188	1.5	0.5	1	3
P5	$\frac{1}{(s+1)(0.2s+1)(0.04s+1)(0.008s+1)}$	1	2.93	1.1	0.02	0.04	2
P6	$\frac{(0.17s+1)^2}{s(s+1)^2(0.028s+1)}$	2	0.186	21.52	0.5	1	2
P7	$\frac{(-2s+1)}{(s+1)^3}$	1	0.19	1.5	0.15	0.3	3
P8	$\frac{1}{s(s+1)^2}$	1	0.278	14.4	0.1	0.2	3
P9	$\frac{e^{-s}}{(s+1)^2}$	2	0.25	1.5	0.5	1	3
P10	$\frac{e^{-s}}{(20s+1)(2s+1)}$	1	3.5	21	0.5	1	2
P11	$\frac{(-s+1)e^{-s}}{(6s+1)(2s+1)}$	1	0.583	7	0.25	0.5	4
P12	$\frac{(6s+1)(3s+1)e^{-0.3s}}{(10s+1)(8s+1)(s+1)}$	2	3.7	1	0.15	0.3	2
P13	$\frac{(2s+1)e^{-0.3s}}{(10s+1)(0.5s+1)}$	3	2.32	4.5	0.15	0.3	2
P14	$\frac{-s+1}{s}$	1	0.385	10.4	0.1	0.2	3
P15	$\frac{-s+1}{(s+1)}$	2	0.385	1	0.1	0.2	2

It shows that the curve estimated from the estimated Box–Jenkins model from the larger value of T_{max} has bigger bias. However, all estimates of jitter effects show the correct trend. The results from Fig. 5(a) indicate that the jitter effect will hurt the control performance if the system has the disturbance model 1 or model 3, this conclusion is proved from the simulation plotted in Fig. 4; and the jitter effect of control performance is relative insignificant if the system has the disturbance model 2. The results for the control loop, P3, in Fig. 5(b) show that the jitter will help the control performance with the disturbance model 1, and will affect the control performance insignificant if the system includes the disturbance model 2, and will hurt the control performance with disturbance model 3.

The same procedure is applied to the remaining 14 systems given in Table 1, and the control performance, and the effect of jitter is summarised in Table 2. The *Theoretical* column lists the MV benchmark without the jitter. From these results we note again that the estimates of performance indices are higher than the true performance indices (consistent with the theory in Section 4.1), and that most of the estimates of the jitter effect on the control performance are corrected (excepting for system P15), reinforcing that the proposed method is reliable.

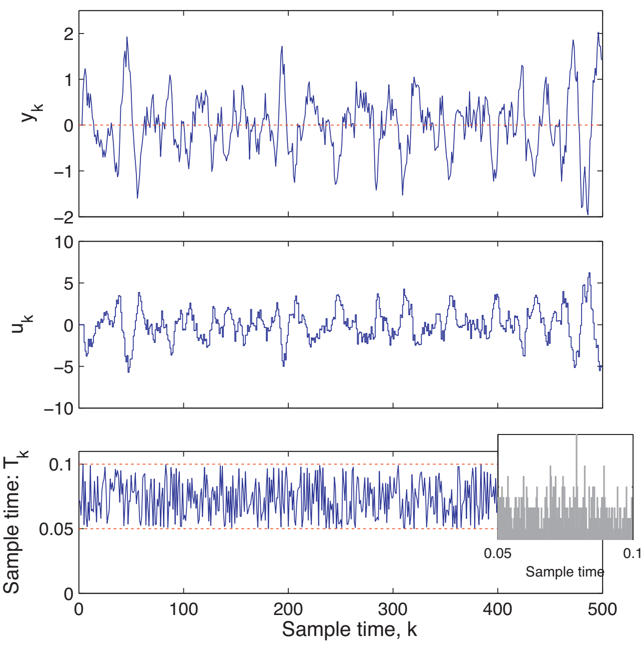

Fig. 3 – The output (upper), input (middle) and instantaneous sampling time (lower) plots for system P1. The PDF of the sample time is given in the insert figure.

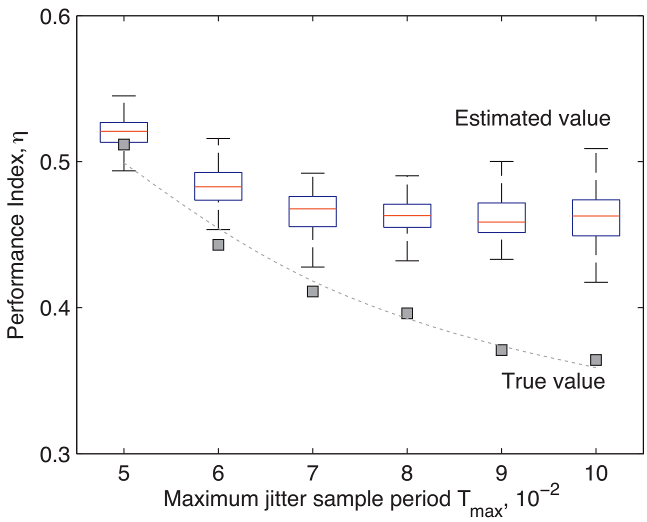

Fig. 4 – Comparative box plots of the quality estimates of the performance index for process model P1.

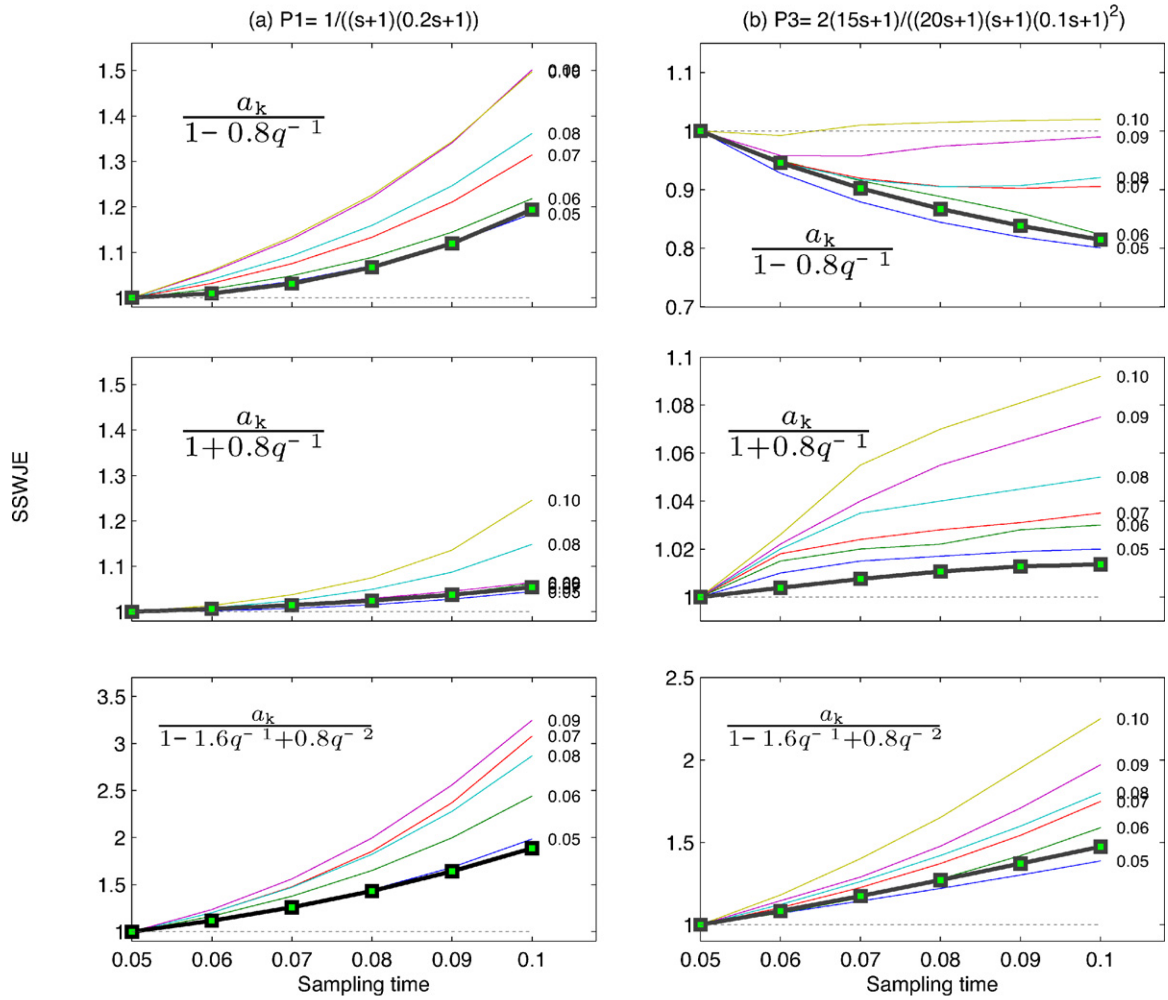

Fig. 5 – The estimates of the jitter effects on the control performance on P1 and P3 using the three noise models.

Table 2 – Summary of the performance assessments of the 15 test plants from Table 1.

Case	Performance index		Jitter effect	
	Theoretical	Estimated	Theoretical	Estimated
P1	0.36	0.45(0.05)	×	×
P2	0.70	0.751(0.05)	√	√
P3	0.475	0.531(0.07)	×	×
P4	0.381	0.398(0.02)	×	×
P5	0.436	0.535(0.04)	–	–
P6	0.648	0.701(0.04)	–	–
P7	0.605	0.624(0.03)	√	√
P8	0.539	0.611(0.05)	×	×
P9	0.781	0.824(0.04)	√	√
P10	0.424	0.513(0.04)	×	×
P11	0.697	0.751(0.05)	√	√
P12	0.01	0.023(0.003)	×	×
P13	0.439	0.512(0.04)	–	–
P14	0.389	0.451(0.04)	×	×
P15	0.268	0.341(0.04)	√	–

√, the jitter helps the control performance.
×, the jitter hurts the control performance.
–, the jitter has minimal affect on the control performance.

6. Conclusions

The contribution of this work is to propose a control performance assessment strategy for linear systems with a sampling jitter problem. The effect of jitter on the control performance is directly estimated from the operating input/output data. Instead of identifying the time-varying ARMA model, we suggest to use the time-invariant ARMA model as an approximator to the time varying ARMA model for the CPA analysis. This strategy is tested on 15 typical processes and the results show that the proposed method is valid for most of the selected processes.

For this study, the sampling jitter is assumed uniformly distributed, although more general distributions of the sampling jitter will be investigated in the future.

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