Rigorously Modelling Steam Utility Systems for Mixed Integer Optimization

Jonathan Currie
Electrical and Electronic Engineering
AUT University
Auckland, New Zealand
Email: jocurrie@aut.ac.nz

David I. Wilson
Industrial Information and Control Centre
AUT University
Auckland, New Zealand
Email: diwilson@aut.ac.nz

***Abstract*—Given that industrial utility systems are essentially large energy converters, it is surprising that they are so often forgotten or ignored when optimizing plant performance. Significant operational savings are possible simply by redistributing steam generation and consumption, without adding extra equipment, and with minimal investment. However due to the discrete nature of a utility system where equipment can switched in and out of service, steam flows redistributed, and zero-flow conditions are normal, the optimizing of utility system requires a rigorous model based on thermodynamics and state-of-the-art numerical algorithms. This paper proposes a mixed integer modelling strategy to approximate a rigorous simulator model, combining regressions from literature, industrial experience and process specific knowledge resulting in a model suitable for optimization. Two case studies are presented to demonstrate the efficiency of the modelling design, a hypothetical three header model with cogeneration and a four header refinery utility system. Both systems are optimized using BONMIN in less than a quarter of a second on a standard desktop PC and result in substantial economic improvements.**

I. Introduction

The management of heat and power in large industrial complexes has always been of considerable interest to those charged with the task of efficiently managing the site. Historically with flat energy prices, and constant production policies, many sites probably found a reasonable optimal operating point eventually, if only by trial and error. However with the development of the smart grid and the subsequent possibilities of for online electricity trading, and the move to flexible production policies means that the efficient use of energy is now crucial, complex and multi-faceted [1].

Industrial utility systems consist of boilers to generate steam which can then be used directly in a different part of the industrial complex, or it can be used as a prime mover, or let down through turbines which produce electricity. For efficiency reasons the condensate is recycled. To heat the condensate, one can burn natural gas, coal or a combination, and the energy in the hot exhaust gas can be further exploited. Finally the utility system can import or export electricity to the local grid. The key point is that large utility systems exhibit a high degree of redundancy which provides flexibility, and the possibility for optimization [2].

Some of the operating conditions in a utility system are externally set such as the pressure levels of the headers, and the user demands, but many other operating conditions are free to be set by operators, or an optimizer. Typical variables include the level of capacity should the boilers operate if at all, whether to purchase electricity or to export it, and where and how much to route the flow of steam. This paper is concerned with the development of tools to model, and algorithms to solve for the best operating conditions for a given situation.

II. Utility System Case Study

This paper will consider the steam utility system based on the system presented in [3]. This work considered a utility system superstructure based on early work by Papoulias and Grossmann in [4] and is representative of the equipment found in typical utility systems. The superstructure has been adapted for use in this work by fixing the equipment structure as we are considering an operational optimization problem as opposed to a design problem. The resulting utility system structure is shown in Figure 1 together with data showing the base case operating condition.

The utility system consists of three steam headers: a high pressure (HP) at 45 bar, a medium pressure (MP) at 17 bar and a low pressure (LP) at 4.5 bar, which are supplied from a HP boiler, high pressure combined cycle gas turbine and heat recovery steam generator (GTG+HRSG), a MP boiler and MP Waste Heat Boiler (WHB). Two condensate levels form the condensation collection system with a 1.4 bar user condensate collection mixer and 0.2 bar condensing turbine exhaust mixer. Two steam turbo generators allow onsite electricity generation, one 6 MW single stage turbo generator, and the other a 5 MW triple stage with condensing third stage. Three back pressure turbines provide shaftwork for process requirements of two compressors and a fan. The required shaftwork is 1.6 MW, 1.2 MW and 0.7 MW for each of the HP-LP, HP-MP and MP-LP turbines. Two condensing turbines complete the system process drivers, supplying 3 MW and 1 MW for the HP and MP to vacuum shaftwork requirements. In addition, all five turbines include parallel electric motors for redundancy.

Three duty based steam users at HP, MP and LP levels represent the heat requirements of an associated chemical process plant. It is assumed in this example that 100% of the steam supplied to the process plant is recovered as saturated condensate and cooled to 110°C. The condensate collected from the process users is mixed together with exhaust from the

978-1-4673-4584-2/12/$31.00

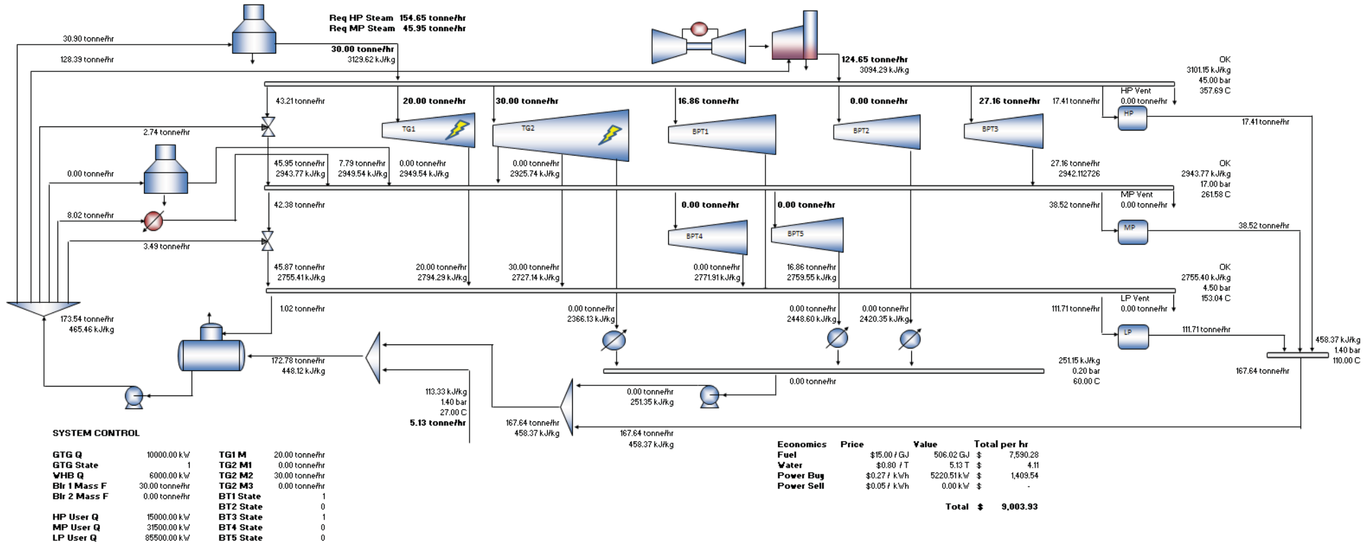

Fig. 1. Case study superstructure together with the base case operating condition.

condensing turbines and completes the condensate recovery of the utility system. Demineralized water is added to complete the system mass balance, and the resulting stream supplied to the deaerator to remove any dissolved gases. This final loop completes the system energy balance, ensuring the model accurately represents the physical system.

A. Base Case

The operational base case is presented in Figure 1 and requires \$9004/hr to run. In this scenario the HP boiler is used to supply 30 tonne/hr of steam, with the remainder made up by the HRSG. A small (15 MW) Gas Turbine Generator (GTG) is used to generate 10 MW of electricity for the site, with the $500°$C GTG exhaust gas supplied to the HRSG. When secondary firing is added this meets the HP steam requirement of 125 tonne/hr. 6 MW of waste heat from the process is used to generate 7.8 tonne/hr of MP steam, completing the steam demand of the system. Process steam demands require 15 MW, 31.5 MW and 85.5 MW of steam from HP, MP and LP respectively, resulting in close to 170 tonne/hr of steam required. To offset the site power requirement of 15 MW, both turbo generators are running with 20 tonne/hr through TG1 and 30 tonne/hr through stage 2 of TG2. However the system electricity balance is still negative, with 4.5 MW of power required from the grid, this is due in part to turbines BT2, BT4 and BT5 switched off and running via the redundant electric motors.

This scenario has been rigorously modelled using the JSteam Excel modelling environment [5]. This includes high speed IAPWS 2007 water and steam thermodynamics [6], Peng-Robinson fuel gas thermodynamics and unit operation models and is validated from our international consulting experience in utility system modelling. This tool will be used to verify all solutions obtained via approximated models in the remainder of this paper. A demo version of this software is available from our website (`www.i2c2.aut.ac.nz`), which apart from the advanced boiler and gas turbine models, includes all remaining functionality.

III. Modelling Steam Utility Systems

When building utility system models one may be tempted to use an existing chemical process simulator such as HYSYS or VMGSim. However, as detailed in previous work [7], steam utility systems have a number of unique features which makes modelling them non-trivial. This is primarily due to the discrete nature of a utility system, where equipment may be switched off completely resulting in a zero mass flow rate. While this value may appear benign, it in fact can cause a range of problems with standard process simulators to the point where the model will fail to solve altogether.

For this reason we developed the JSteam modelling framework with the specific aim of rigorously modelling utility systems where entire units may be switched on and off without numerical problems. This also addresses other utility system modelling requirements such as detailed boiler and gas turbine models. The framework is implemented as an optimized C++ library with .NET, Excel and MATLAB interfaces. The library contains a number of unit operation models from industry and academia and allows both full and part load modelling of a range of equipment models using accurate thermodynamics. However while this library provides a detailed framework for *simulating* utility systems, it is not suitable for optimizing these systems. We previously explored the substantial difference between optimizing simulator and purpose built utility models in [8] and demonstrated that even on small systems time differences of nearly two orders of magnitude were observed by the solver. This is a well known problem in the optimization field, and thus new models must be developed which are suitable for optimization.

A. Boiler Model

The utility boiler is fundamental to the operation of the utility system and thus must be modelled as accurately as possible. Fortunately when viewed as a relationship between fuel mass flow and steam mass flow, this relationship is virtually linear. This result has been verified by examining a range of boiler sizes and operating conditions, including steam temperature and pressure, and varying input fuel gas compositions, and is also detailed in [9]. However rather than obtaining an approximate model for a range of boilers and operating conditions, because we are considering operational optimization, we can linearize about our current operating point.

To illustrate, the HP boiler from our case study is approximated. The boiler is generating 369°C, 45 bar superheated steam with a continuous blow down ratio of 0.03. We assume a constant Boiler Feed Water (BFW) Enthalpy of 465 kJ/kg, as well as a fuel temperature of 27°C and air temperature (after air pre-heater) of 80°C. The boiler is operated assuming a minimum stack temperature of 250°C, and a minimum inlet excess O_2 molar fraction of 0.15. As is common in refinery operations the boiler is run on natural gas, and in this instance composed of (molar fractions) 0.7289 methane, 0.2589 ethane and 0.012 nitrogen.

The resulting linear model is:

$$M_{\text{fuel}} = 0.0607 M_{\text{steam}} \tag{1}$$

where as expected there is no offset, indicating that when no steam is being produced, no fuel gas is being consumed. In reality, all boilers have a minimum steam production, below which they are not operated. However to keep the model smooth and thus differentiable, the linear fit is kept and an additional binary variable is added to the model. Using the Big-M strategy to define either-or constraints, the following linear inequalities are added to the model, assuming a minimum boiler steam flow of 10 tonne/hr, and a maximum of 250 tonne/hr:

$$M_{\text{steam}} - 300y \leq 0 \tag{2}$$

$$-M_{\text{steam}} + 300y \leq 290 \tag{3}$$

where *y* is a binary variable. When *y* is 0 (boiler off), the first constraint is imposed, and when *y* is 1, the second constraint is imposed which implements the minimum flow constraint.

B. Turbo Generator Model

A steam turbo generator is a steam turbine connected to an electric generator, which can be used to supply site power demands. However as it could run between no-load and full-load in an optimization run, a part-load model must be created. The full load model is given by the standard thermodynamic relations of an isentropic steam turbine:

$$H_{\text{out}} = H_{\text{in}} - \eta \Delta H_{\text{isen}} \tag{4}$$

$$Q_{\text{shaft}} = \eta M_{\text{steam}} \left(H_{\text{in}} - H_{\text{out}} \right) \tag{5}$$

where H_{in} is inlet Enthalpy, M_{steam} is the steam mass flow through the turbine, η is the isentropic efficiency of the turbine and ΔH_{isen} is the isentropic enthalpy drop across the turbine. There are two properties which must be calculated, η which is a function of turbine size and load, and ΔH_{isen} which is a function of operating conditions of the utility system.

In order to determine η for a particular turbine we used the work of Peterson and Mann [10]. Their well cited work examined the operating conditions of a large range of steam turbines and presented data illustrating the maximum efficiency of a steam turbine given its rated shaftwork. This data was used to build a general relationship shown in Figure 2.

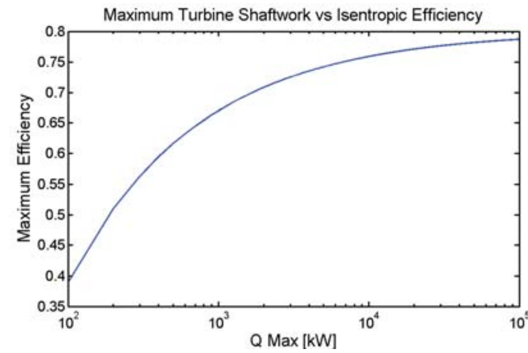

Fig. 2. Regressed fit for determining maximum isentropic efficiency of a steam turbine based on rated shaftwork.

Once the maximum efficiency of a turbine had been established, the part-load efficiency can be determined using the relationships described in [11], [12]. This allows us to generate an expression which relates the maximum turbine efficiency, steam flow and header conditions to the actual turbine efficiency.

It is worth noting that even though the efficiency expression is nonlinear, the resulting curve is approximately linear between 40% and 100% load, which was also observed in [9]. However for our turbine model we have opted to keep the nonlinear terms to better model the turbine behaviour at zero-flow. If TG1 were to be modelled using a Willan's line approximation then the output power when the turbine was off would be negative.

C. Back Pressure Turbine Model

The back pressure turbine model is based on a simplified version from Section III-B. As the turbine shaftwork is fixed according to the load being driven, the turbine efficiency can also be approximately fixed, resulting in a much simpler model. Therefore the only term which requires calculation is the isentropic enthalpy drop across the turbine, which is a constant if the header enthalpy is fixed, or approximated by a quadratic function of the header enthalpy if it is variable. As the turbine can be switched on or off, depending whether the load is steam driven or electric motor driven, a binary variable is added for each turbine. The binary variable is used to implement a linear equality constraint which either constrains the turbine shaftwork to the required shaftwork if it is on, or constrains it to 0 if it is off.

IV. Operational Optimization

In order to optimize the utility system an optimizer specific model must be built, utilizing the models described in the previous section. To do this the system in Figure 1 must be written out in terms of explicit mass and energy balances, including the described models where appropriate. There are several packages that allow this type of modelling, however we have opted to develop our own modelling system within MATLAB. This is primarily based on our development of the OPTI Toolbox [13], which provides an umbrella over a suite of academic and open-source optimizers from the OR community. This software is free to download and is in active development with a large following of international users.

A. Objective Function

The cost function used with this work considers the three main costs associated with running a utility system: fuel gas consumption, electricity costs and demineralized water consumption. The most complex function here is the site electricity balance, described in terms of power generated (Q_{gen}) and power required (Q_{req}) from the system equipment

$$Q_{\text{gen}} = Q_{\text{GTG}} + \sum_{n=1}^{2} Q_{\text{TGn}} \tag{6}$$

$$Q_{\text{req}} = \sum_{n=1}^{5} (1 - b_n) Q_{\text{BTn}} + \sum_{n=1}^{2} Q_{\text{PUMPn}} \tag{7}$$

$$Q_{\text{total}} = Q_{\text{gen}} - Q_{\text{req}} - Q_{\text{site}} \tag{8}$$

The electricity balance is combined into the full objective function

$$j = \alpha \sum_{n=1}^{2} FuelM_{\text{BLRn}} + \beta Q_{\text{total}} + \gamma M_{\text{water}} \tag{9}$$

where α is the cost per tonne of fuel gas, β is the cost of either buying or selling electricity, depending on the sign of Q_{total} and γ is the cost per tonne of demineralized water. Binary variables b_n indicate whether back pressure turbine *n* is switched on or off.

B. Model Implementation in MATLAB

To best leverage the equation orientated nature of optimizer model, we have implemented it in a customized modelling language embedded in MATLAB. In this way the model can be described as general mathematical equations (versus blackbox functions), and the Symbolic Toolbox can be used to generate exact, analytical derivatives of the model equations. An added benefit of using a symbolic manipulator is that the equations are automatically simplified, reducing model size without required user input. For the base case system the modelling engine requires only 3 seconds to parse, simplify and generate exact first and second derivatives of the entire model.

The resulting MINLP model consists of 77 variables of which 9 are binary, 70 constraints of which 43 are linear and 27 are nonlinear, and 154 bounds. The modelling engine has also automatically identified linear, quadratic and nonlinear constraints and variables, allowing it to generate an efficient sparse representation of the constraint Jacobian (255 non-zero terms) and model Hessian (42 non-zero terms). The linear and nonlinear information is also automatically propagated through to the MINLP solver in order to maximize the solver efficiency.

C. Base Case Optimal Operation

The symbolic model can be automatically converted into a OPTI model, and solved using one of free MINLP solvers available with OPTI. In this work we are using the gradient based, convex MINLP solver BONMIN [14], which utilizes IPOPT as the relaxed solver. Using the Quesada & Grossman outer approximation algorithm the optimizer returns in 0.12s with an optimized minimum of \$7805/hr. The optimizer has chosen to shutdown Boiler 1 as it is more efficient to generate HP steam using the GTG + HRSG. As the price of selling excess electricity is not attractive, the GTG is fired so that maximum steam production of the HRSG is equal to the steam demand. BONMIN has also chosen to switch BT4 on, further reducing electricity import required by the site.

The optimizer has also made two interesting, non-trivial decisions. The first is that the optimal operating condition has actually increased HP steam demand by 6 tonne/hr, however the net effect is increased shaftwork can be generated by steam via the installed turbines, reducing electricity demand and thus the hourly cost by nearly \$1200/hr. The second is the decision not to increase mass flow through TG1, to avoid the letdown mass flow via the MP-LP desuperheater. By inspection it would seem increasing mass flow TG1, and reducing it from TG2 Stage 1 would reduce the wasted letdown flow, and decrease operating cost. However this is where modelling accurately both the turbine exhaust enthalpy, as well as varying header enthalpies is important. By increasing the flow through TG1 the outlet enthalpy begins to drop (due to the turbine efficiency increasing with load). As TG1 and TG2 both supply a large amount of steam to the LP header, they directly affect the header temperature. The optimizer has constrained the LP header enthalpy to be greater than saturated steam, thus a delicate balance between the two turbines exist, and it is in fact more economical to let down steam, rather than cool the header.

D. Optimization Subject to Varying Input Conditions

In order to verify that the proposed modelling and optimization strategy is robust, we will examine several alternative operating conditions which could commonly be encountered when operating the proposed system. A summary of results is presented in Table I together with key system variables for each scenario. The results presented are consist of the optimized conditions inserted into the rigorous thermodynamic model.

1) Attractive Electricity Generation Pricing: It is not uncommon in a deregulated electricity market that the price of electricity can spike when electricity supply is interrupted or the demand exceeds current generation running capacity. In this scenario utility systems with cogeneration facilities

TABLE I
SUMMARY OF OPTIMIZED RESULTS FOR EACH SCENARIO.

Scenario	1	2	3
Base Case Cost	\$13337	\$6445	\$7770
Optimized Cost	\$1243	\$4143	\$6597
Solver Time	0.12s	0.25s	0.15s
Solver Nodes	5	2	6
M_{HRSG}	119.3 t/h	65.8 t/h	130.2 t/h
M_{HPBLR}	72.7 t/h	0 t/h	0 t/h
M_{MPBLR}	0 t/h	0 t/h	0 t/h
Q_{GTG}	15 MW	15 MW	11.84 MW
Q_{TG1}	6 MW	4.1 MW	3.7 MW
Q_{TG2}	5 MW	3.55 MW	5 MW
Q_{total}	10.6 MW	0 MW	0 MW

which are able to generate surplus electricity can both assist in stabilizing the grid, as well potentially make a profit exporting electricity. However, reacting quickly to this change, as well as in the most economical way is not always obvious. In this scenario the price of buying electricity has spiked to \$1.10 kWh, and selling electricity to \$0.80 kWh. Given the operating conditions of base case in Figure 1 this results in a hourly cost of \$13337.

This scenario was optimized using BONMIN in 0.1s resulting in a hourly cost of \$1243, representing a significant OPEX saving. This has been achieved by maximizing the available electricity generation of the site, with all turbo generators running at full load, as well as the GTG. All turbines are also run via steam, resulting in a surplus of 10.5MW of electricity available to sell back to the grid. HP steam demand has increased nearly 40 tonne/hr to 192 tonne/hr, however the steam demand is split via the HP boiler and HRSG, with the boiler supplying 73 tonne/hr of superheated steam.

2) Decreased Process Heat Demand: During normal operation of a process plant routine maintenance, equipment failure, or simply the end of a batch operation can all cause large reductions in the heat requirements from the utility system. During this period of low user duties finding the optimum generation and distribution of steam can be non-trivial. In this scenario the user heating duties have dropped to 0 MW, 10 MW and 35.5 MW for the HP, MP and LP users respectively. The hourly cost of this scenario assuming operating conditions as in the base case is \$6445.

The optimized operating point is obtained in 0.25s resulting in an hourly cost of \$4143. As expected, BONMIN has opted to substantially reduce steam production by shutting down the HP boiler and reducing the HRSG steam production to the demand of 66 tonne/hr. The very low steam demand is the minimum for the utility system in order to keep supplying the required heat to the process users, while shutting down all back pressure turbines. TG1 and TG2 take up the role of letdown, generating electricity while letting HP steam down to the MP and LP levels. The optimizer has also managed to balance the site electricity demands by running the GTG at full load, and optimizing the mass flows through the turbo generators, requiring no electricity to be purchased from the grid.

3) Increased MP Waste Heat Boiler Duty: The final scenario considered in this work is an increase of waste heat available for steam generation from the process. The utility system contains a MP Waste Heat Boiler (WHB) which is currently utilizing 6 MW of available process heat to generate 7.8 tonne/hr of 264°C superheated steam to the MP header. In this final scenario the available waste heat has increased to 30 MW, allowing 39 tonne/hr of steam to be supplied to the MP header. Given the base case operating conditions this results in a hourly cost of \$7770.

Once optimized using BONMIN the optimum operating point is obtained in 0.15s with an hourly cost of \$6597. As with the previous scenario, the optimizer has shut down the HP boiler, and the HP steam demand is now met solely by the HRSG. As the MP header now has a surplus of steam, BT4 (MP - LP) is switched on and BT3 (HP - MP) switched off. Mass flow through stage two of TG2 is also maximized to generate the most electricity possible from TG2, while keeping the LP header temperature just above saturated steam. The optimizer has also balanced the electricity demand of the system, with no power being imported or exported.

V. APPLICATION TO AN INDUSTRIAL UTILITY SYSTEM

This modelling framework has been applied to an actual industrial utility system which our research team was previously involved with. To preserve sensitive economic information, we have made slight changes to the structure shown in Figure 3 while still maintaining the key characteristics and complexity of the industrial problem. The system contains four steam pressure levels, three small, identical, boilers and nineteen process drivers. As with most typical utility systems, the drivers are connected in parallel forming two groups, HP - LP and MP - LP drivers. Desuperheaters between the headers provide steam to the lower pressure levels where required. Three duty based steam users form the main purpose of this utility system providing heat to the associated chemical process plant.

Due to legacy issues, no cogeneration equipment exists and thus all power required by the system must be bought from the national grid. The optimization of this system is therefore a driver selection problem, matching steam turbines and electric motors to shaftwork demands in the most economical manner.

This utility system is modelled in MATLAB using our symbolic interface to create an efficient MINLP representation of the plant. The MINLP model consists of 103 variables, of which 19 are binary, 75 constraints of which 36 are linear and 39 are nonlinear, and 206 bounds. The full model, including all analytical derivatives, is built in 3 seconds. Using BONMIN the base case is optimized in 0.15s resulting in \$86/hr OPEX saving reducing the hourly running cost to \$4029. This was achieved by switching any steam turbine under 50kW off and importing electricity to run the required load, as the small turbines are simply too inefficient given the current electricity price. While this result is not as significant as in the examples provided so far, real savings in the range of 1-2% are common in the industrial literature [15], [16] for operational optimization of utility systems, which matches the 2% saving obtained in this example.

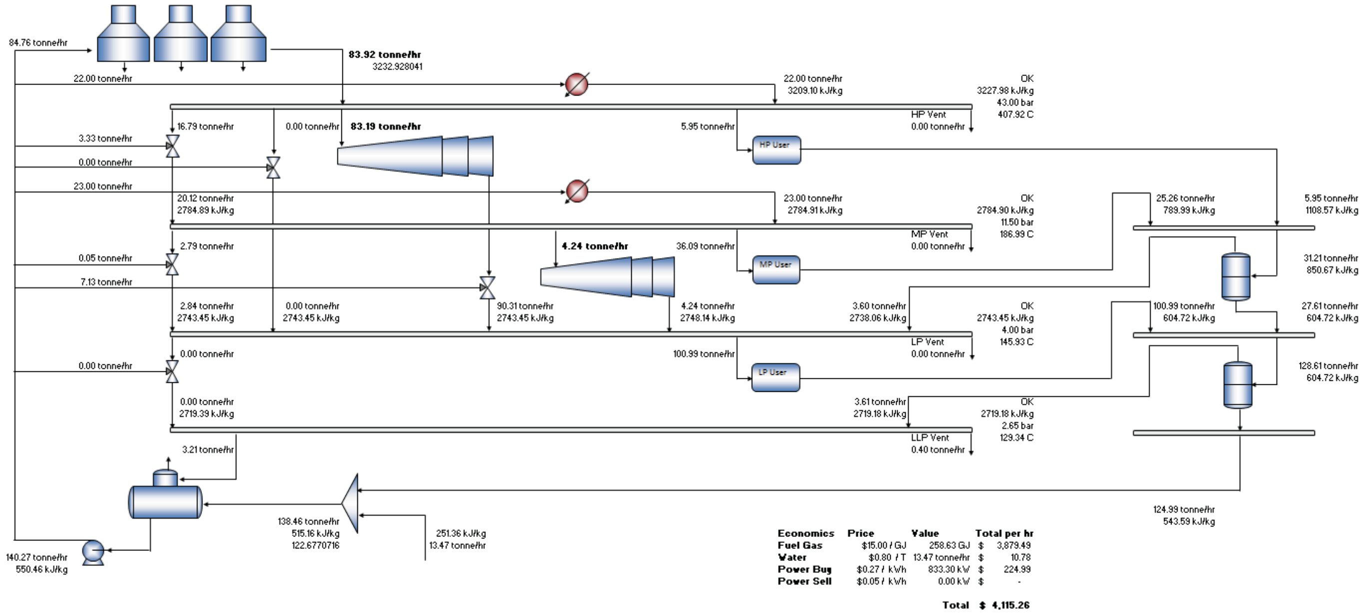

Fig. 3. Industrial utility system base case.

VI. Conclusion

This paper has detailed the operational optimization of two utility systems; one hypothetical from literature but containing a representative selection of equipment, and the other a real industrial system from a petrochemical refinery. Using MATLAB, purpose built optimization models could be built and validated against the rigorous models within the JSteam framework, and then optimized using the, open-source mixed integer optimization package BONMIN. In a variety of operating scenarios all models converged in less than a quarter of a second and all resulted in significant OPEX savings. Using the proposed symbolic mixed integer modelling framework it allowed exact derivatives to be generated, greatly aiding the speed and accuracy of the optimizer in finding local solutions.

Acknowledgment

Financial support to this project from the Industrial Information and Control Centre, Faculty of Engineering, AUT University and The University of Auckland, New Zealand is gratefully acknowledged.

References

[1] M. H. Agha, R. Thery, G. Hetreux, A. Hait, and J. M. Le Lann, "Integrated production and utility system approach for optimizing industrial unit operations," *Energy*, vol. 35, no. 2, pp. 611–627, 2010.

[2] Aguilar, O. and Perry, S. J. and Kim, J. K. and Smith, R., "Design and optimization of flexible utility systems subject to variable conditions: Part 2: Methodology and applications," *Chemical Engineering Research and Design*, vol. 85, no. 8, pp. 1149–1168, 2007.

[3] J. C. Bruno, F. Fernandez, F. Castells, and I. E. Grossmann, "A Rigorous MINLP Model for the Optimal Synthesis and Operation of Utility Plants," *Chemical Engineering Research and Design*, vol. 76, no. 3, pp. 246–258, 1998.

[4] S. A. Papoulias and I. E. Grossmann, "A structural optimization approach in process synthesis–I: Utility systems," *Computers & Chemical Engineering*, vol. 7, no. 6, pp. 695–706, 1983.

[5] J. Currie, "JSteam Excel Add In: Utility System Thermodynamics and Modelling in Excel," available from `www.i2c2.aut.ac.nz`.

[6] IAPWS, "Revised Release on the IAWPS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam," p. 49, 2007.

[7] J. Currie, D. I. Wilson, N. Depree, B. R. Young, S. Azmani, and L. Lukman, "Steam Utility Systems are not 'Business as Usual' for Chemical Process Simulators," in *American Institute of Chemical Engineers (AIChE) Spring Meeting*, Chicago, USA, 2011.

[8] J. Currie and D. I. Wilson, "The Efficient Modelling of Steam Utility Systems (Submitted)," in *Australian and New Zealand Annual Chemical Engineering Conference, Chemeca*, Wellington, New Zealand, 2012.

[9] O. Aguilar, S. J. Perry, J. K. Kim, and R. Smith, "Design and optimization of flexible utility systems subject to variable conditions: Part 1: Modelling framework," *Chemical Engineering Research and Design*, vol. 85, no. 8, pp. 1136–1148, 2007.

[10] J. F. Peterson and W. L. Mann, "Steam system design: How it evolves," *Chemical Engineering*, vol. 92, no. 21, pp. 62–74, 1985.

[11] S. P. Mavromatis and A. C. Kokossis, "Conceptual optimisation of utility networks for operational variations–I: Targets and level optimisation," *Chemical Engineering Science*, vol. 53, no. 8, pp. 1585–1608, 1998.

[12] P. S. Varbanov, S. Doyle, and R. Smith, "Modelling and optimization of utility systems," *Chemical Engineering Research and Design*, vol. 82, no. 5, pp. 561–578, 2004.

[13] J. Currie and Wilson, "OPTI: Lowering the Barrier Between Open Source Optimizers and the Industrial MATLAB User," in *Foundations of Computer-Aided Process Operations (FOCAPO) 2012*, Savannah, Georgia, 2011.

[14] P. Bonami, L. T. Biegler, A. R. Conn, G. Cornuejols, I. E. Grossmann, C. D. Laird, J. Lee, A. Lodi, F. Margot, and A. Waechter, "An Algorithmic Framework for Convex Mixed Integer Nonlinear Programs," *Discrete Optimization*, vol. 5, no. 2, pp. 186–204, 2008.

[15] D. Fernandez-Polanco, A. Eastwood, and N. Knight, "Real-time utility system optimization," in *Twenty-Sixth Industrial Energy Technology Conference*, Houston, Texas, 2004.

[16] G. J. Fien, "Make the most of it," *The Chemical Engineer*, 2008.