

# Control Performance Assessment for a class of Nonlinear Multivariable Systems

Wei Yu & Brent R. Young\*

Chemical & Materials Engineering  
The University of Auckland, New Zealand

David I. Wilson

Electrical & Electronic Engineering  
Auckland University of Technology, New Zealand

## Overview

Crucial control loops in industrial applications are often multivariable and nonlinear due to the plant, the transducers, the actuators, or even in some cases the controllers themselves.

Both these issues mean that it is awkward to assess control loop performance using standard tools.

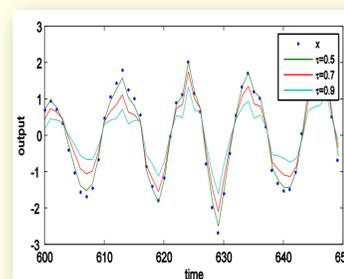
This study extends control performance assessment tools to nonlinear MIMO systems with additive linear disturbances and where the nonlinearity is in the form of valve stiction.

Valve stiction is a very common nonlinear problem in industrial processes, and causes biased estimates of control performance indices.

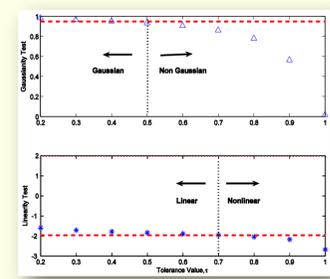
## Performance Index Estimation

- Use a B-spline to remove the nonlinearity
- The optimal B-spline is determined by the Hinich Gaussianity & linearity test
- The residual time-series between the B-spline curve and the output  $y_t$  can be used to estimate the performance index

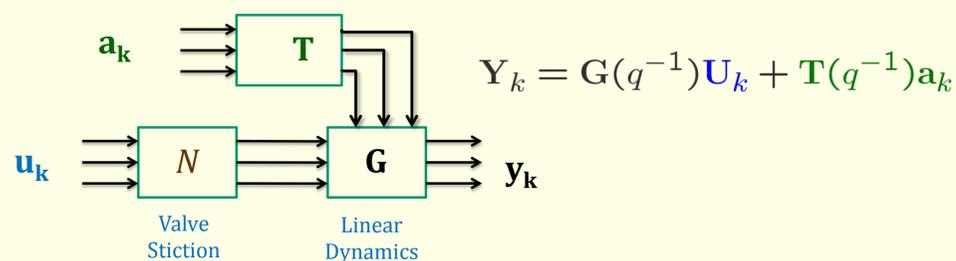
B-spline fitting curve



Hinich test



## Performance Index for MIMO System with Valve Stiction



The overall performance index (PI) for a MIMO system can be defined as:

$$\eta = \frac{\text{trace}(\Sigma_{mv})}{\text{trace}(\Sigma_y)}$$

where  $\Sigma_y$  is the actual covariance matrix of the process estimated from the process data,  $\text{trace}(\Sigma_{mv})$  is the Minimum Variance Performance Lower Bound (MVPLB), and  $\Sigma_{mv}$  is the covariance matrix under minimum variance control. This latter matrix is defined as,

$$\Sigma_{mv} \stackrel{\text{def}}{=} \sum_{i=0}^{d-1} F_i \Sigma_e F_i^T$$

where  $\Sigma_e$  is the covariance matrix of the error,  $d$  is the time delay, and  $F_i$  are the infinite impulse response matrices of the process which can be calculated by solving,

$$q^{-d} D G_{cl} = \sum_{i=0}^{\infty} F_i q^{-i}$$

where  $D$  denotes the interactor matrix which is assumed to be known, and  $G_{cl}$  is the closed-loop model.

## Simulations

$$G = \begin{bmatrix} \frac{q^{-(d-1)}}{1-0.4q^{-1}} & \frac{0.7q^{-d}}{1-0.1q^{-1}} \\ \frac{0.3q^{-(d-2)}}{1-0.1q^{-1}} & \frac{q^{-(d-1)}}{1-0.8q^{-1}} \end{bmatrix}, \quad T = \begin{bmatrix} \frac{1}{1-0.5q^{-1}} & \frac{-0.6q^{-1}}{1-0.6q^{-1}} \\ \frac{0.5q^{-1}}{1-0.7q^{-1}} & \frac{1}{1-0.8q^{-1}} \end{bmatrix}$$

Deviations of PIs are in range of 12% to 33% which may be misleading if the output nonlinearity is not removed. With the proposed method, deviations are reduced from double to single digits.

## Conclusions

- We extended the estimate of MVPLB from SISO linear systems with a valve stiction problem to MIMO linear systems
- The proposed method reduces large over-estimates of the actual performance indices.

## References

- [1] Yu, W., Wilson, D.I. & Young, B.R. (2010), Nonlinear control performance assessment in the presence of valve stiction, *Journal of Process Control* **20**(6):754-761.
- [2] Harris, T.J. & Yu, W. (2007), Controller assessment of a class of nonlinear systems, *Journal of Process Control*, **17**: 607-619.

## \*Contacts

E-mail: [b.young@auckland.ac.nz](mailto:b.young@auckland.ac.nz)  
Website: [www.auckland.ac.nz/i2c2](http://www.auckland.ac.nz/i2c2)