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Control Performance Assessment for a class of Nonlinear Multivariable Systems

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Abstract

Control loops of importance in industrial applications are often multivariable and exhibit nonlinear dynamics stemming either from the plant, the transducers, the actuators, or even in some cases the controllers themselves. Both these issues, the multivariable nature of the control loop and the nonlinearities, mean that it is awkward to reliably assess the control loops performance using standard tools. This study extends control performance assessment tools to nonlinear multi-input multi-output (MIMO) systems. To make the problem tractable, we restrict our nonlinear MIMO system structure to be a system with additive linear disturbances and where the nonlinearity is in the form of valve stiction.

Keywords: Control performance assessment, Multivariable systems, Valve stiction.

1. Introduction

Control performance assessment, or CPA, is the name given to a broad range of strategies intended to maintain operational efficiency of industrial control systems. CPA includes the automated diagnosis of under-performing control loops, the establishment of control benchmarks and recommending solutions. Much of the established practice assumes linear SISO plant models disturbed by Gaussian noise and has enjoyed success in many industries as noted in [1-3].

However a strictly linear SISO analysis can give misleading results when faced with the inevitable nonlinearities found in many industrial control loops. These nonlinearities could be due to nonlinearities in the actuators, the transducers, or even the plant itself. Estimates of the minimum variance performance lower bound (MVPLB), which is a key component when establishing a benchmark to quantify the controlled performance, and the subsequent performance index using linear CPA techniques will be distorted by these nonlinearities [4-8]. For example, Yu *et al.* [6] show that one tends to overestimate the performance index for linear systems with an additive linear disturbance affected by valve stiction when using linear CPA techniques which can lead to a false sense of security. To deal with this situation, recent research has proposed several methods to extend CPA for nonlinear systems.

In this paper, we will extend nonlinear CPA tools, specific for the minimum variance performance bound, to multivariable linear systems where the nonlinearity is caused by valve stiction. Control valve stiction is widely recognised as a common industrial problem [9], prompting [10-13] to investigate ways to diagnose the issue, while [14] attempted to regress parametric stiction models and thereby indirectly quantify the stiction problem. Valve stiction is not just an important operational copncern, it is also illustrative of a challenging class of nonlinear processes. Stiction is a dynamic nonlinearity, non-differentiable and often involves characteristics that change gradually over time. For these reasons stiction makes a good case study for nonlinear CPA techniques.

The layout of the paper is as follows. In Section 2, the problem statement and model including valve stiction is introduced. Section 3 describes the MVPLB for MIMO systems and methods which can be used to estimate the minimum variance lower bound with valve stiction. In Section 4, a simulation example is used to illustrate the proposed method. This is followed by a discussion and conclusions highlighting both the limitations and potential of the alternative methods.

2. Process Description

Consider a square mulitvariable process with *m* inputs and *m* outputs that can be expressed in the following linear representation,

$$\boldsymbol{Y}_t = \boldsymbol{G}(q^{-1})\boldsymbol{U}_t + \boldsymbol{T}(q^{-1})\boldsymbol{a}_t \tag{1}$$

where $\boldsymbol{Y}_t$, $\boldsymbol{U}_t$ and $\boldsymbol{a}_t$ are output, input and noise vectors of appropriate dimensions. The noise vector $\boldsymbol{a}_t$ is further assumed to be white noise with zero mean and covariance matrix $\mathrm{Var}(\boldsymbol{a}_t)=\boldsymbol{\Sigma}_a$. The transfer functions matrices $\boldsymbol{G}(q^{-1})$ and $\boldsymbol{T}(q^{-1})$ are proper, rational in the backshift operator q^{-1}. $\boldsymbol{T}(q^{-1})$ is a rational realization of the disturbance spectrum with the standard assumptions that $\boldsymbol{T}(q^{-1}=0)=\boldsymbol{I}$ and is minimum phase.

The common process nonlinearity afflicting control valves known as 'stiction' is modeled by the two-parameter model proposed in [15] where the valve behaviour is characterised by a valve dead-band plus stick band, *s*, and a valve slip jump, *j*.

In this work, we are not overly concerned with *excessive* stiction as that is relatively easily recognised (perhaps by its tell-tale triangular periodic waveform or one of the many strategies outlined in [13], but rather in the cases where the valve stiction is relatively small, and hence easily overlooked, but still insidious. After all, when the output data exhibits excessive and obvious oscillation, the only sensible option is to first service the valve, and only then perform a CPA.

3. Performance measures for multivariable linear system with valve stiction

For MIMO system CPA, the interactor matrix was defined as a generalization of the delay encountered in univariate situations. With a full knowledge of the interactor matrix, a MIMO minimum variance performance benchmark was proposed by Harris *et al.* [16, 17], Huang et al. [18]. Estimation of the interactor matrix has been discussed in several papers [19, 20]. Although it is possible to construct the interactor matrix, computation of the benchmark is still much more difficult [21]. Several approaches for performance assessment that do not require knowledge of the interactor matrix were proposed in [21-24].

The overall performance index for a MIMO system described by Equation (1) can be defined as:

$$\eta = \frac{trace(\tilde{\boldsymbol{\Sigma}}_{mv})}{trace(\tilde{\boldsymbol{\Sigma}}_{y})} \tag{2}$$

where $\tilde{\Sigma}_y$ is the actual covariance matrix of the process estimated from the process data, $trace(\tilde{\Sigma}_{mv})$ is the MVPLB, and $\tilde{\Sigma}_{mv}$ is the covariance matrix under minimum variance control. This latter matrix is defined as

$$\tilde{\Sigma}_{mv} = \sum_{i=0}^{d-1} \boldsymbol{F}_i \Sigma_e \boldsymbol{F}_i^T \tag{3}$$

where Σ_e is the covariance matrix of the error, d is the time delay, and $\boldsymbol{F}_i$ are the infinite impulse response matrices of the process which can be calculated by solving

$$q^{-d} \boldsymbol{D}\boldsymbol{G}_{cl} = \sum_{i=0}^{\infty} \boldsymbol{F}_i q^{-i} \tag{4}$$

where $\boldsymbol{D}$ denotes the interactor matrix which is assumed to be known in this paper, and $\boldsymbol{G}_{cl}$ is the closed-loop model.

3.1. Nonlinear removal

One output of a linear system with valve stiction can be decomposed into two distinct parts: a linear part from the additive disturbance and the nonlinear part from the feedback loop. Figure 1 illustrates this decomposition showing the measured output, the undisturbed plant output and the disturbance signal.

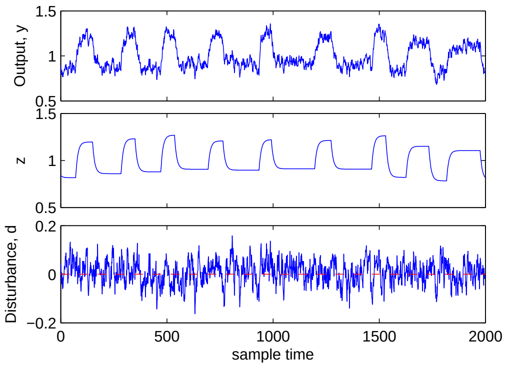

Figure 1: Decomposing the output into linear and nonlinear components. Top: output, y, middle: the undisturbed plant output, z, and lower: disturbance

A heuristic way to remove the nonlinearity caused by the valve stiction for the purposes of subsequently establishing the performance of the controller is by using some sort of local smoother such as a spline.

For this application we have employed a smoothing B-spline, with a single smoothing parameter, τ, as implemented by the Matlab function **spaps** in the spline toolbox. A large tolerance value will give a smoother approximate curve. A suitable tolerance is established by finding the largest smoothing parameter τ where the residual between the output y and the spline is both Gaussian and linear. This is an iterative search procedure using the statistical tests for Gaussianity and linearity described next.

A statistical test for determining whether an observed stationary time series is linear was proposed by Hinich [25] and a summary of tests for nonlinearity can be found in [26]. The Hinich test is nonparametric and robust and has previously been applied for the diagnosis and detection of valve stiction in [15]. If the series is Gaussian, its skewness is zero and its squared bicoherence is also zero. If however, the series is linear, its skewness and the squared bicoherence are constant, but not necessarily zero. These two properties are the basis for the Hinich Gaussianity and linearity tests.

4. Simulations

We will use the linear two by two example discussed by Huang et al. [21] with plant and disturbance transfer matrices,

$$\boldsymbol{G}(q^{-1}) = \begin{bmatrix} \frac{q^{-(d-1)}}{1-0.4q^{-1}} & \frac{0.7q^{-d}}{1-0.1q^{-1}} \\ \frac{0.3q^{-(d-2)}}{1-0.1q^{-1}} & \frac{q^{-(d-1)}}{1-0.8q^{-1}} \end{bmatrix} \quad \boldsymbol{T}(q^{-1}) = \begin{bmatrix} \frac{1}{1-0.5q^{-1}} & \frac{-0.6q^{-1}}{1-0.6q^{-1}} \\ \frac{0.5q^{-1}}{1-0.7q^{-1}} & \frac{1}{1-0.8q^{-1}} \end{bmatrix} \tag{5}$$

and where both inputs pass through a valve stiction nonlinear element. The white noise input $\boldsymbol{a}_t$ is a two-dimensional white noise sequence with the covariance matrix $\boldsymbol{\Sigma}_a=\boldsymbol{I}$.

The values of the MVPLB for several d values are estimated based on the two different closed-loop system identification methods: i) directly estimate the closed-loop system using time series analysis (system identification) from the raw output data (MV_1 is used to denote the MVPLB estimated from this method); and ii) remove the nonlinearity from the output data caused by the valve stiction and then estimate the closed-loop system (MV_2 is used to denote the MVPLB estimated from this method). The results are listed in Table 1 (MV_d is the theoretical MVLB value).

Table 1: Estimates of the MVLB

	d				
	2	3	4	5	6
MV_d	5.22	6.71	6.45	7.96	7.67
MV_1	5.88	7.84	7.8	9.40	10.19
Deviation %	12.6	16.8	17.8	18.1	32.9
MV_2	5.65	7.22	7.10	8.67	8.78
Deviation %	8.2	6.7	8.3	8.9	14.5

From Table 1, we observe that the deviations of the MVPLB estimates increase when the time delay d increases if the output nonlinearity is not removed, and the deviations are in range 12% to 33% which may be misleading. The estimates of the MVPLB, MV_2, based on output nonlinearity removal, show a deviation reduction from double digits to single digit (except for d=6). Comparing the deviations of the MVPLB estimates from these two methods, we can conclude that the output nonlinearity removal can help improve the MVPLB estimate.

5. Conclusions

In this paper, we extend the estimate of MVPLB from SISO linear systems with a valve stiction problem to MIMO linear systems. We have defined the performance index in Eqn. (2) using only the diagonal terms, and in this instance we have assumed that the interactor matrix is known. Consequently the $\boldsymbol{F}_i$ can solved using Diophantine equation. From the simulation results, it is shown in this case that simple ignorance of the valve stiction nonlinearity will cause a relatively large over-estimate of the actual value. The proposed method reduces this gap. Future work will try to reduce the dependency upon the strategy to prior knowledge of the interactor matrix.

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